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THE REDUCED ORDER MODEL PROBLEM IN DISTRIBUTED PARAMETER SYSTEMS ADAPTIVE IDENTIFICATION AND CONTROL: AN INTERIM REPORT

for NASA Grant NAG-I-7

by

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NOTICE

I. Recent Progress Summary

The research sponsored by NASA Langley Research Center Grant NAG-I-7 is addressing the reduced order model (ROM) problem in distributed parameter systems (DPS) adaptive identification and control by focusing on the development and evaluation of an adaptive controller applicable to the active stabilization of a DPS, such as a large flexible spacecraft, given a necessarily reduced-order expansion approximation model structure. The adaptive control strategy chosen for investigation [1] was an outgrowth of earlier NASA-sponsored efforts to combine the modal expansion endemic to the large flexible spacecraft community [2] and self-tuning [3], which is the most widely accepted lumped-parameter system (LPS) adaptive control strategy. The annular momentum control device (AMCD) [4] [5] was used as the test example.

The difficulty encountered in the initial work [6]-[8] supported by NAG-I-7 was the deleterious effect of control and observation spillover. Two interpretations of this problem, which is unavoidable in application of existing LPS adaptive control schemes to DPS expansion descriptions, have emerged. In [6] and [8] a discrete-time state model of a generic class of DPS

$$V_N(k+1) = \phi_N V_N(k) + \phi_{NR} V_R(k) + E_N f(k) \tag{1}$$

$$V_R(k+1) = \phi_{RN} V_N(k) + \phi_R V_R(k) + E_R f(k) \tag{2}$$

$$y(k) = C_N V_N(k) + C_R V_R(k) \tag{3}$$

is formulated where the subscript N denotes the ROM quantities and R the residual model quantities. Assuming that the projection generating the ROM is done into the modal subspace yields $\phi_{NR} = \phi_{RN} = \underline{0}$. This state-space interpretation and attempts at its adaptive control resulted in the

following observations: (i) If there is no observation spillover, i.e. $C_R = \underline{0}$, and no model error, i.e. $\emptyset_{NR} = \underline{0}$, then an indirect adaptive control strategy, e.g. [9], can be globally stable [8]. (ii) Unless $C_R = \underline{0}$ and $\emptyset_{NR} = \underline{0}$ (1) is only a quasi autoregressive moving-average (ARMA) and not a true ARMA model [6]. The alternate interpretation of the ROM effects presented in [7] is based on consideration of two types of identifier strategies. A DPS sufficiently accurately described by a very high dimension modal expansion

$$d(\theta,t) = \sum_{j=0}^{N} W_j(t)\ \emptyset_j(s), \tag{4}$$

where the $\emptyset_j$ are the spatial mode shapes and W_j their amplitude time histories, can be approximated by either a subset of the N modes of (4)

$$d_R(\theta,t) = \sum_{j=0}^{M} W_j(t)\ \emptyset_j(s) \tag{5}$$

or by a selection of $\hat{W}_j$ to minimize some measure of $d-\hat{d}$ with

$$\hat{d}(\theta,t) = \sum_{j=0}^{M} \hat{W}_j(t)\ \emptyset_j(s). \tag{6}$$

In neither (5) nor (6) must the M+1 modes used in the ROM be the first M+1 in the full N+1 mode expansion. The observations made in [7] were: (i) Since only d can be measured some signal processing is necessary to yield d_R in order to solve (5) for the nonphysical W_j needed in the individual modal dynamics identifiers and controllers. (ii) The fitting of $\hat{d}$ to d by selection of the $\hat{W}_j$ may not yield adequately accurate values of the corresponding W_j for stable modal identification and control. This latter fact was emphatically demonstrated in [7] by simulated use of (6) and resulting DPS instability. Observation spillover can be interpreted as the source of difficulty in extracting d_R from d or in having $\hat{W}_j$

closely approximate W_j if $\hat{d}$ is fit to d. Unfortunately the apparent usefulness of proposed approaches to observation spillover reduction is severely limited [6]-[8].

II. Future Plans

Since observation spillover seems to defy complete removal, the question in applying available LPS adaptive control techniques to a ROM of a DPS appears to be their limits of tolerance to observation spillover. Unfortunately, the LPS adaptive control literature contains very little generalizable insight into the reduced-order adaptive control problem. In fact the nonadaptive LPS reduced-order control problem is currently an open research question.

Therefore in order to pursue the original objective of DPS adaptive controller development by judicious application of existing LPS adaptive control schemes, the thrust of the future research under NASA Grant NAG-I-7 is being shifted to LPS reduced-order adaptive controller (ROAC) studies. The insight gained on this subset of the DPS adaptive control problem will hopefully prove transferable. Admittedly the ROM effects are slightly different. In LPS ROAC the problem is the lower order of the dynamic model selected; while in DPS ROAC the orders of the individual modal dynamics are assumed accurate but the number of modes is insufficient. However the source of difficulty in both can be viewed as spillover.

Consider a discrete-time LPS described by the transfer function

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_1}{z-a_1} + \frac{\varepsilon b_2}{z-a_2} \ . \tag{7}$$

In a state description similar to (1)-(3) ε would correspond to C_R and due to the partial fraction expansion $\emptyset_{NR}$ would equal $\underline{0}$ for the ROM

$$\hat{y}(k) = \hat{a}\ y(k-1) + \hat{b}\ u(k-1). \tag{8}$$

Assuming a stable control objective of

$$G(z) = \frac{X(z)}{R(z)} = \frac{d}{z-c} \Rightarrow x(k) = cx(k-1) + dr(k-1) \tag{9}$$

the control effort

$$u(k-1) = f_1 r(k-1) + f_2\, y(k-1) \tag{10}$$

with

$$f_1 = \frac{d}{\hat{b}} \tag{11}$$

$$f_2 = \frac{c-\hat{a}}{\hat{b}} \tag{12}$$

would adequately control (7) if $\varepsilon = 0$ and $\hat{b}$ and $\hat{a}$ had converged to b_1 and a_1 respectively.

Four generic possibilities will be tested for tolerance to nonzero ε for this simplistic reduced-order model reference adaptive control (MRAC) task: (i) equation error based indirect adaptive control (EI), (ii) equation error based direct adaptive control (ED), (iii) output error based indirect adaptive control (OI), and (iv) output error based direct adaptive control (OD). The EI approach uses equation error identification [10] of the plant parameters in a self-tuning [3] solution of the stated MRAC problem. The EO technique is based on an input matching [11] [12] solution to the MRAC problem. The OI approach uses HARF [13] as an output error plant parameter identifier in conjunction with the separation (or "certainty equivalent") assumption of self-tuning. The OD scheme is based on an output error identification interpretation [14] of the MRAC problem with a minor modification in the control law so the adaptive controller can be put in the form of the error system in [15] and proven convergent. Though these approaches are asymptotically equivalent for $\varepsilon = 0$,

variations are anticipated in this ROAC test. The complete algorithms for each case are listed below:

Plant:
$$y(k) = (a_1+a_2)y(k-1) - (a_1a_2)y(k-2) + (b_1+b_2\varepsilon)u(k-1) - (b_1a_2+\varepsilon b_2a_1)u(k-2) \quad (13)$$

EI:
$$\hat{y}(k) = \hat{a}(k)\ y(k-1) + \hat{b}(k)u(k-1) \quad (14)$$

$$\hat{a}(k+1) = \hat{a}(k) + \frac{\mu y(k-1)[y(k)-\hat{y}(k)]}{1+\mu y^2(k-1)+\rho u^2(k-1)} \quad (15)$$

$$\hat{b}(k+1) = \hat{b}(k) + \frac{\rho u(k-1)[y(k)-\hat{y}(k)]}{1+\mu y^2(k-1)+\rho u^2(k-1)} \quad (16)$$

$$\hat{f}_1(k) = \frac{d}{\hat{b}(k+1)} \quad (17)$$

$$\hat{f}_2(k) = \frac{c-\hat{a}(k+1)}{\hat{b}(k+1)} \quad (18)$$

$$u(k) = \hat{f}_1(k)\ r(k) + \hat{f}_2(k)\ y(k) \quad (19)$$

ED:
$$\hat{f}_1(k) = \hat{f}_1(k-1) + \frac{\mu_1 r(k-1)[dr(k-1)+cy(k-1)-y(k)]}{(\operatorname{sgn} b)\ (\max|b|)[1+\mu_1 r^2(k-1)+\mu_2 y^2(k-1)]} \quad (20)$$

$$\hat{f}_2(k) = \hat{f}_2(k-1) + \frac{\mu_2 y(k-1)[dr(k-1)+cy(k-1)-y(k)]}{(\operatorname{sgn} b)\ (\max|b|)[1+\mu_1 r^2(k-1)+\mu_2 y^2(k-1)]} \quad (21)$$

$$u(k) = \hat{f}_1(k)r(k) + \hat{f}_2(k)\ y(k) \quad (22)$$

OI:
$$\hat{y}(k) = \hat{a}(k)\ z(k-1) + \hat{b}(k)\ u(k-1) \quad (23)$$

$$\hat{a}(k+1) = \hat{a}(k) + \frac{\mu z(k-1)[y(k)-\hat{y}(k) + q(y(k-1) - z(k-1))]}{1 + \mu z^2(k-1) + \rho u^2(k-1)} \quad (24)$$

$$\hat{b}(k+1) = \hat{b}(k) + \frac{\rho u(k-1)[y(k)-\hat{y}(k) + q(y(k-1) - z(k-1))]}{1 + \mu z^2(k-1) + \rho u^2(k-1)} \quad (25)$$

$$z(k) = \hat{a}(k+1)\ z(k-1) + \hat{b}(k+1)\ u(k-1) \quad (26)$$

$$\hat{f}_1(k) = \frac{d}{\hat{b}(k+1)} \tag{27}$$

$$\hat{f}_2(k) = \frac{c-\hat{a}(k+1)}{\hat{b}(k+1)} \tag{28}$$

$$u(k) = \hat{f}_1(k)\ r(k) + \hat{f}_2(k)\ y(k) \tag{29}$$

OD: $$x(k) = c\ x(k-1) + d\ r(k-1) \tag{30}$$

$$\hat{f}_1(k) = \hat{f}_1(k-1) + \mu_1 r(k-1)[x(k) - y(k) + q(x(k-1) - y(k-1))] \tag{31}$$

$$\hat{f}_2(k) = \hat{f}_2(k-1) + \mu_2 y(k-1)[x(k) - y(k) + q(x(k-1) - y(k-1))] \tag{32}$$

$$\begin{aligned} u(k) = {} & \hat{f}_1(k)\ r(k) + \hat{f}_2 y(k) \\ & + [1 + \mu_1 r^2(k-1)+\mu_2 y^2(k-1)][x(k)-y(k)+q(x(k-1)-y(k-1))] \end{aligned} \tag{33}$$

The sensitivity of each scheme to ε will be compared via determination of $\sum_k (x(k)-y(k))^2$, $\max_k (x(k)-y(k))^2$, $\sum_k u^2(k)$, and $\max_k u^2(k)$ for various a_i and b_i. Clearly the relative damping of the modeled and unmodeled portions of the plant should prove important. The most basic ROM rule-of-thumb is the ability to neglect modes with settling times $\frac{1}{10}$ or less of the dominant modes. However, earlier research [6]-[8] indicates that the smallness of ε will also be important.

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IV. Recent Presentations

"The Reduced Order Model Problem in Distributed Parameter Systems Adaptive Identification and Control," The University of Newcastle, Newcastle, New South Wales, Australia, February 13, 1980.

V. Recent Sponsored Publications

1. M. J. Balas and C. R. Johnson, Jr., "Adaptive control of distributed parameter systems: The ultimate reduced order problem" (Invited paper), Proc. 18th IEEE Conf. on Dec. and Control, Ft. Lauderdale, FL, pp. 1013-1017, December 1979.

2. C. R. Johnson, Jr., "On adaptive modal control of large flexible spacecraft," to appear in J. Guidance and Control, vol. 3, no. 4, July-August 1980.

3. M. J. Balas and C. R. Johnson, Jr., "Toward adaptive control of large space structures," to appear in Applications of Adaptive Control, eds. K. S. Narendra and R. V. Monopoli. New York: Academic Press, 1980.

ADAPTIVE CONTROL OF DISTRIBUTED PARAMETER SYSTEMS: THE ULTIMATE REDUCED-ORDER PROBLEM

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Abstract

For many distributed parameter systems (DPS), such as highly mechanically flexible structures, it is essential to provide stable on-board/on-line adaptive control in the presence of poorly known system parameters. However, such a controller must be based on a reduced-order model of the DPS and spillover from the unmodeled residuals can deteriorate the performance of the controller and, in some cases, defeat the whole purpose of the adaptive control.

In this paper, we investigate direct and indirect adaptive controllers based on reduced-order models of a DPS and point out the mechanisms whereby spillover can upset the stability of adaptive controllers. We present some conditions under which the adaptive controllers remain stable in closed-loop with the actual DPS and point out certain generic problems that must be overcome for successful operation of adaptive DPS control.

1.0 Introduction

A great abundance of adaptive control schemes exists for linear, lumped parameter, small scale systems [1], whose system parameters are poorly known, and the application of adaptive control is being seriously considered (and, in some cases, hotly debated) in a variety of areas [2]. The existing adaptive control methods can be divided into two (not unfriendly) camps: direct schemes, where the available control parameters are directly adjusted (adapted) to improve the overall system performance, e.g. [3]-[6], or indirect (self-tuning) schemes, where the plant parameters are estimated by some reasonably fast system identification scheme and the control commands are generated from these parameter estimates as though they were the actual values, e.g. [7] [9]. The direct schemes are usually model reference adaptive, i.e. the controlled system (plant) is forced to behave like a model system which has the desired performance properties, such as transient response (pole locations).

As G. Kreisselmeier succinctly put it: "adaptive control trades plant uncertainty for uncertainty in the closed-loop system behavior"; the stability of a linear system which uses an adaptive controller is often in question because the closed-loop system will be highly nonlinear during adaption. Direct (or model reference) adaptive schemes have achieved a great amount of success in producing globally stable closed-loop behavior under certain technical restrictions on the plant [6]. Global stability of certain indirect schemes using adaptive estimators has been shown in [8], [9] with fewer restrictions on the plant replaced by the need for addition of "sufficiently rich" external test signals to the control commands. Presently, all stable adaptive schemes are restricted to scalar (or at best, multi-input, single-output) systems; the stability for general multivariable systems remains an important unsolved problem in adaptive control. Another fundamental restriction is that the plant must be of known finite dimension, and this dimension must be small enough to meet computational constraints for the on-line use of the particular adaptive scheme.

The abundance of adaptive control schemes is overwhelming and an understanding of the interrelationships and structural commonality of these methods is desperately needed; see [6], [10]-[11] for progress in that direction.

This paper addresses the problems that appear when even the most well behaved (i.e. stable) adaptive schemes for lumped parameter systems (LPS) are applied to linear distributed parameter systems (DPS), i.e. systems whose behavior must be modeled by partial differential equations (PDE). It is natural to want to make maximum use of the existing body of LPS adaptive control theory in the new context of DPS and it is obvious that "something" will go wrong - according to a well known folk theorem: if it can go wrong, it will. We want to point out some of the basic mechanisms through which things can go wrong for adaptive control of DPS. Our viewpoint is shaped by experience with application of DPS control to large aerospace structures, e.g. [12]-[14], and this paper is an expansion of some of the topics presented in [15]; however, what we have to say here is in the broader context of DPS where applications include such diverse topics as flutter suppression for aircraft [16] and actively controlled civil engineering structures, e.g. tall buildings and long bridges, subject to high winds or earthquakes [17].

The most fundamental generic problem for adaptive DPS control is that DPS are infinite dimensional and any implementable adaptive scheme must be based on a reduced-order model (ROM) of the original system. Therefore, the adaptive controller order must always be smaller than that of the plant no matter how much on-line computational capability is available to implement the controller. This violates the hypotheses of all the existing global stability results for direct or indirect adaptive control. In our estimation, this makes adaptive DPS control the ultimate reduced-order problem and, until this problem is solved (or circumvented), the benefits of adaptive control will be denied to what may be its most needy customer-DPS applications. Preliminary attempts at adaptive control for some specific DPS have been made in [18]-[24].

2.0 DPS Description

The class of DPS considered here may be described by the following:

$$\begin{cases} v_t = Av + Bf;\ v(0) = v_0 \\ y = Cv \end{cases} \tag{2.1}$$

where, for each positive time t, the (possibly vector-valued) system state $v(t)$ is in H, an appropriate Hilbert space with inner product $(\cdot,\cdot)$ and corresponding norm $\|\cdot\|$. The operator A is a differential operator which is time-invariant, has domain $D(A)$ dense in H, and generates a C_0 semigroup $U(t)$ on H. The control vector f is $M\times 1$ and the sensor output vector is $P\times 1$ and rank $B=M$ and rank $C=P$; this means that a finite number of control actuators (M) and sensors (P) are used.

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The semigroup $U(t)$ is dissipative in the sense that

$$||U(t)|| \le M_0 e^{-\varepsilon t} \quad , t \ge 0 \tag{2.2}$$

where $M_0 \ge 1$ and $\varepsilon \ge 0$ (when $\varepsilon=0$, we assume $M_0=1$). This means that the energy in $v(t)$, i.e. $||v(t)||^2$, is conserved or dissipated. It is known [25], [26] that if (2.2) does not hold with $\varepsilon > 0$ then it cannot be made so even with ideal state feedback:

$$f = Gv \tag{2.3}$$

because B has finite rank. Therefore, we will usually assume $\varepsilon > 0$ because most physical systems have some dissipative mechansims in them; however, ε may be extremely small (as it is in large aerospace structures). Equation (2.1) can represent a large class of DPS including mechanically flexible structures such as aircraft and spacecraft.

3.0 Reduced-Order Modeling of DPS

Even though (2.3) may be the ideal feedback control law, the full (infinite dimensional) state v is rarely available. Implementable controllers for most DPS must be based on on-board/on-line control computers which process the P sensor outputs y and produce the M control commands f. Hence, control design must be based on a reduced-order model (ROM) of the DPS in (2.1).

ROM's are produced by projecting the system (2.1) onto some appropriate finite dimensional subspace H_N of H; such projections often are orthogonal but they need not be. Let $v_N=Pv$ and $v_R=Qv$ where P is the projection of H onto H_N and Q is the projection of H onto the residual subspace (the subspace complementary to H_N in H) and, from (2.1), we obtain:

$$\dot{v}_N = A_N v_N + A_{NR} v_R + B_N f \tag{3.1}$$

$$\dot{v}_R = A_{RN} v_N + A_R v_R + B_R f \tag{3.2}$$

$$y = C_N v_N + C_R v_R \tag{3.3}$$

where $A_N=PAP$, $A_{NR}=PAQ$, $B_N=PB$, etc. The terms $B_R f$ and $C_R v_R$ are called control and observation spillover, respectively; the terms $A_{NR}v_R$ and $A_{RN}v_N$ are called model error. The ROM is obtained from (3.1) and (3.3) with $A_{NR}=0$ and $C_R=0$:

$$\begin{cases} \dot{v}_N = A_N v_N + B_N f; \; v_N(0) = P v_0 \\ y = C_N v_N \end{cases} \tag{3.4}$$

The ROM state v_N and the residual state v_R form the full state $v = v_N + v_R$; when the projections are orthogonal,

$$||v||^2 = ||v_N||^2 + ||v_R||^2 \tag{3.5}$$

All controllers designed on the ROM (3.4) must be evaluated in closed-loop with the full system (3.1)-(3.3), i.e. the effects of the residuals v_R through model error and spillover must be considered. In some cases these effects are sufficiently small that they can be ignored and the DPS is controlled quite well from a ROM-based controller; most times this is not the case-Murphy's law, again!

The choice of subspace H_N and type of projection is often a matter of clever design and insight into the specific DPS. However, there are some obvious candidates, e.g. Galerkin approximation using any appropriate spline basis [27]. Another good candidate is the modal subspace:

$$H_N = \text{sp}\ \{\phi_1, \ . \ . \ . \ , \phi_N\}$$

with

$$A\phi_k = \lambda_k \phi_k \tag{3.6}$$

where ϕ_k are the orthonormal mode shapes (eigenfunctions) of A corresponding to the eigenvalues λ_k. Such mode shapes are available when the operator A is normal and has compact resolvent; this is the case for most mechanically flexible structures modeled by (2.1). Note that, in many cases, the exact mode shapes may exist but not be known. When the modal subspace is used, the projection is assumed orthogonal and the model error terms A_{NR} and A_{RN} are zero; however, the spillover terms depend on the actuator-sensor locations and the mode shapes and need not be zero. Other choices of subspace and method of projection could be selected in an attempt to reduce the model error and spillover terms, e.g. [28].

4.0 Nonadaptive DPS Control

When the parameters of the ROM are completely known, a linear controller can be designed:

$$f + H_{11}\, y + H_{12}\, z \qquad \begin{aligned} f &= H_{11}y + H_{12}z \\ \dot{z} &= H_{21}y + H_{22}z \end{aligned} \tag{4.1}$$

where the controller gains H_{ij} are obtained from the ROM parameters (A_N, B_N, C_N) and the desired performance. Once the controller is designed, the effect of the residuals must be analyzed using (4.1) in closed-loop with (3.1)-(3.3). Energy bounds have been produced in [29] to predict the effect of spillover and model error on the stability and performance of the reduced-order nonadaptive control (4.1) with the DPS (2.1).

Examples of controlled flexible structures have been produced [13], [30] where the spillover terms caused instabilities in the residuals even though the ROM (3.4) in closed-loop with the controller (4.1) would have been stable.

Digitally implemented controllers would be based on discrete-time versions of the DPS (2.1). One such version can be obtained by using the uniform time step Δt with a constant control command $f(k)$ over the interval $(k-1)\Delta t \le t \le k\Delta t$:

$$\begin{cases} v(k+1) = \Phi v(k) + H\, f(k) \\ y(k) = C\, v(k) \end{cases} \tag{4.2}$$

where $\Phi = U(\Delta t)$ and $H = \int_0^{\Delta t} U(\tau)\, B\, d\tau$. Other versions could be obtained for example with nonuniform time steps. When the ROM procedure of Sec. 3.0 is used, the discrete-time versions of (3.1)-(3.3) become

$$v_N(k+1) = \Phi_N\, v_N(k) + \Phi_{NR}\, v_R(k) + H_N\, f(k) \tag{4.3}$$

$$v_R(k+1) = \Phi_{RN} v_N(k) + \Phi_R\, v_R(k) + H_R\, f(k) \tag{4.4}$$

$$y(k) = C_N v_N(k) + C_R\, v_R(k) \tag{4.5}$$

5.0 Adaptive Control of DPS

The natural approach for adaptive control of DPS seems to be

(a) choose a nice ROM;
(b) use your favorite lumped parameter adaptive control scheme;
(c) design the adaptive controller as though the ROM were the actual system to be controlled;
(d) analyze the closed-loop behavior of the adaptive controller with the actual DPS.

Often, step (d) is omitted. This can be disasterous as shown by the flexible structure example in [31].

Even a globally stable adaptive scheme may prove to be unstable when used on the full DPS instead of the ROM; this happens because of the effect of spillover and model error. The indirect adaptive control scheme of [8] uses an adaptive observer and would be globally stable on the ROM under certain technical restrictions. However, if it were designed on the ROM (3.4) but used on the full DPS (2.1), it would not necessarily be stable; the mechanism whereby spillover enters the adaptive control scheme and spoils the stability proof is displayed in [15]. This is not a failure of the method of [8]; it is a failure to satisfy the hypothesis of the stability result of [8]-every DPS will fail to satisfy this hypothesis unless the observation spillover and the model error term Δ_{NR} are both zero (see Theorem in [15]). The model error term is always zero when the exact mode shapes are used for the ROM but the observation spillover need not be zero even then. Hence some means of compensation, such as an adaptive prefilter [15] or an adaptive version of the orthogonal filter [32], might be used to try to eliminate observation spillover, but this has not been fully studied.

ARMA-Gettin'

The stable direct adaptive control schemes depend on an Auto-Regressive Moving Average (ARMA) representation of the plant in discrete-time:

$$y(k+N) = \sum_{r=1}^{N} \alpha_r y(k+r-1) + \sum_{r=1}^{N} \beta_r f(k+r-1) \tag{5.1}$$

for some N and appropriate matrices α_r, β_r. What the ARMA says is that, after N time steps, the present output is only related to the past N outputs and inputs. Existence of an ARMA is directly related to the finite dimensionality of the plant (N is usually that dimension) and is obtained using the Cayley-Hamilton theorem for matrices. For DPS, only a "quasi-ARMA" can exist. From the Appendix, we obtain the quasi-ARMA for the DPS (4.2) or (4.3)-(4.5):

$$y(k+N) = \sum_{r=1}^{N} \alpha_r y(k+r-1) + \sum_{r=1}^{N} \Gamma_r H_n f(k+r-1) + R(k) \tag{5.2}$$

where

$$R(k) = C_R v_R(k+N) + \sum_{r=1}^{N} \Delta_r v_R(k+r-1)$$

$$\Delta_r = \Gamma_r \phi_{NR} - \alpha_r C_R$$

and Γ_r is defined in the Appendix. Since $R(k)=0$ when $C_R=0$ and $\Delta_r=0$, we have the following result:

Theorem: When the observation spillover (C_R) and the model error term (ϕ_{NR}) are both zero, the quasi-ARMA (5.2) is a true ARMA for the DPS (4.2) and any stable adaptive scheme based on this ARMA will be globally stable when used in closed-loop with the actual DPS (4.2).

This result for direct schemes is analogous to the one for indirect schemes in [15]. Both results require that observation spillover be eliminated somehow before it reaches the adaptive control logic. This is not easy to do in general!

6.0 Generic Problems in Adaptive of DPS Control

The most crucial problem of adaptive control of DPS is that the plant is infinite dimensional and, consequently, the adaptive controller must be based on a low-order model of the DPS in order to be implemented with an on-line/on-board computer. However, any controller based on a reduced-order model (ROM) must operate in close-loop with the actual DPS; it interacts not only with the ROM but also with the residual subsystem through the spillover and model error terms. This contributes the following generic difficulties for stable adaptive controllers:

(1) interaction of the residual subsystem with the adaptive controller may negate the stabilizing properties of the controller unless observation spillover can be completely removed before it reaches the adaptive control logic;
(2) the nonlinear nature of the adaptation mechanisms in these controllers substantially aggravates the residual interaction problem (note elimination of control spillover alone does not guarantee stability as it does in the nonadaptive case);
(3) compensation for spillover (and model error) often requires knowledge of the very plant parameters that are poorly known-a vicious circle;
(4) the adaptation mechanism may shift the closed-loop frequencies around (i.e. the closed-loop system is time-varying as well as nonlinear); this may counteract any benefits from prefiltering to remove observation spillover.

In addition to the above, the following also tend to spoil the known stability results for adaptive controllers:

(a) DPS control problems are often non-minimum phase due to noncollocated actuators and sensors; this presents a stability problem for direct schemes but not for indirect ones;
(b) DPS control must often be done with more than one actuator and sensor; conversion of multivariable systems to scalar systems via output feedback can be used to overcome the lack of a multivariable adaptive control but it introduces some new problems (i.e. it may destabilize the residuals) [15];
(c) indirect schemes need sufficient excitation from external test signals; however, these signals may substantially excite the residual subsystem.

Some robust or adaptive way to counteract observation spillover must be found in order to make stable adaptive control of DPS possible. Even then, observation spillover will be present to some degree; consequently, global stability may be too much to ask of these controllers. Development of spillover bounds, along the lines of those for nonadaptive controllers, may be able to give some idea of the regions of stability for successful operation of adaptive DPS control; such development is now in progress.

Appendix. Derivation of the Quasi-ARMA for a DPS

In this appendix, we will derive the quasi-ARMA (5.2) for the DPS (4.3)-(4.5).

From (4.3), define h(k) so that

$$v_N(k+1) = \phi_N \, v_N(k) + h(k) \qquad (A.1)$$

Then, for any non-negative interger L,

$$v_N(k+L) = \phi_N^L \, v_N(k) + \sum_{r=1}^{L} \phi_N^{L-r} \, h(k+r-1) \qquad (A.2)$$

Note that the summation in (A.2) must be zero when L=0.

From the Cayley-Hamilton Theorem,

$$\phi_N^N = \sum_{r=1}^{N} \alpha_r \, \phi_N^{r-1} \qquad (A.3)$$

Take L=N in (A.2) and use (A.3) to obtain

$$v_N(k+N) = \sum_{r=1}^{N} \{\alpha_r(\phi_N^{r-1} v_N(k)) + \phi_N^{N-r} \, h(k+r-1)\} \qquad (A.4)$$

From (A.2) with L=r-1, obtain

$$\phi_N^{r-1} \, v_N(k) = v_N(k+r-1) - \sum_{j=1}^{r-1} \phi_N^{r-1-j} \, h(k+j-1) \qquad (A.5)$$

Use (A.5) in (A.4) and obtain

$$v_N(k+N) = \sum_{r=1}^{N} \alpha_r v_n(k+r-1) + Q(k) \qquad (A.6)$$

where

$$Q(k) = \phi_N^{N-1} \, h(k) + \sum_{r=2}^{N} \{\phi_N^{N-r} \, h(k+r-1) - \alpha_r \sum_{j=1}^{r-1} \phi_N^{r-1-j} \, h(k+j-1)\}$$

$$= \{\phi_N^{N-1} - (\alpha_2 + \alpha_3 \, \phi_N + \dots + \alpha_N \, \phi_N^{N-2})\} \, h(k)$$

$$+ \{\phi_N^{N-2} - (\alpha_3 + \alpha_4 \, \phi_N + \dots + \alpha_N \, \phi_N^{N-3})\} \, h(k+1)$$

$$\vdots$$

$$+ \{\phi_N - \alpha_N\} \, h(k+N-2) + h(k+N-1)$$

Therefore, define Γ_r by

$$Q(k) = \sum_{r=1}^{N} \Gamma_r \, h(k+r-1) \qquad (A.7)$$

and use this in (A.6) to obtain

$$v_N(k+N) = \sum_{r=1}^{N} \{\alpha_r v_n(k+r-1) + \Gamma_r \, h(k+r-1)\} \qquad (A.8)$$

Use (A.8) in (4.5) to obtain

$$y(k+N) = \sum_{r=1}^{N} \alpha_\ell \, \{y(k+r-1) - C_R v_R(k+r-1)\}$$

$$+ \sum_{r=1}^{N} \Gamma_r h \, (k+r-1) + C_R v_R(k+N) \qquad (A.9)$$

Rearranging terms in (A.9) and using the definition of h(k) in (A.1), obtain

$$y(k+N) = \sum_{r=1}^{N} \alpha_r \, y(k+r-1) + \sum_{r=1}^{N} \Gamma_r H_N \, f(k+r-1) + R(k) \qquad (A.10)$$

where

$$R(k) = C_R v_R(k+N) + \sum_{r=1}^{N} \Delta_r v_R(k+r-1)$$

and

$$\Delta_r = \Gamma_r \, \phi_{NR} - \alpha_r C_R \, .$$

This is the desired quasi-ARMA.

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ON ADAPTIVE MODAL CONTROL OF LARGE FLEXIBLE SPACECRAFT

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Abstract

A recently developed strategy for adaptive sampled-data control of distributed parameter systems based on a plant modal expansion description and modal simultaneous identification and regulation algorithms is presented with frequent reference to the annular momentum control device (AMCD) test example. The requirement of observation spillover reduction, which is especially crucial to the proposed adaptive control strategy, is addressed.

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NOTATION

a, b	modal differential equation parameters
A	number of actuators
C	number of modes controlled
d	distributed parameter system deflection
d_R	reduced order modal model deflection
$\hat{d}$	approximate reduced order model deflection
e	identification error
f	actuator forces
F	modal forces
M	number of modes in approximate expansion
N	number of modes in accurate expansion
s	spatial variable
S	number of sensors
t	time
T	sample period
W	modal amplitudes
α, β	modal difference equation parameters
γ, δ	modal controller parameters
Γ	AMCD spin rate
θ	ring particle angle with respect to reference particle
λ	desired discrete characteristic equation coefficients
μ, ρ	adaptive identifier step-size weights

σ	actuator reference frame angles
ϕ	partial differential equation expansion spatial eigenvectors
ω	oscillatory modal amplitude time frequency
Ω	set of distributed system particles

Introduction

The control of large flexible spacecraft has become an active research and development topic, as demonstrated by a recent survey[1]. Adaptive control of distributed parameter systems (DPS) is also an emerging research concern[2-5]. Since large flexible spacecraft are acknowledged to be described by partial differential equations with uncertain, i.e. a priori indeterminable, parameters, such structures, requiring increasingly stringent shape and attitude regulation, are prime candidates for application of DPS adaptive control strategies[6].

The objective of the present work is to provide a real-time simultaneous identification and control strategy applicable to DPS in general and large flexible spacecraft in particular. The real-time computation objective prompts the use of modal expansion descriptions of DPS to permit some parallel computation. The constantly activated adaptability of simultaneous identification and control is needed, for example, for adequate control of poorly behaved DPS, such as very lightly damped, large flexible spacecraft, during inadequately predictable plant parameter changes due to operating condition variations. Toward this goal, this paper improves a previously proposed strategy[4] with further attention to a step-by-step adaptive controller development procedure and its consequences. The original idea[4] was to combine a truncated modal expansion description used in flexible spacecraft control[1] with a simultaneous identification and control (also termed self-tuning[7]) adaption strategy to regulate the lumped parameter modal

amplitude descriptions. The procedure of the next section also mentions the possibility of direct rather than indirect modal control parameter adaption, such as via model reference adaptive control[8]. The outlined procedure also reacts further to the special problems spillover[9] creates for an adaptive implementation. This strategy is followed in the third section to develop an adaptive regulator of the linearized, out-of-plane deflection of a spinning annular momentum control device (AMCD), a candidate for large flexible spacecraft control[4,10,11]. Simulations of this application are presented in the fourth section. This example is also used in the fifth section to illustrate the difficult modal observation spillover reduction problem. An enlarged framework for adaptive control of distributed parameter systems, especially flexible spacecraft, is overviewed in the conclusion. Unfortunately, achievement of the grandiose objective stated at the start of this paragraph is only (possibly) begun in this paper.

A Distributed Parameter System Adaptive Control Strategy

For DPS describable by

$$d(s,\ t) = \sum_{j=0}^{\infty} W_j(t)\ \phi_j(s), \tag{1}$$

where d is a vector-valued function of spatial location s and time t, e.g. displacement for flexible spacecraft, $\phi_j(s)$ an orthogonal expansion basis of shape eigenfunctions, and $W_j(t)$ the amplitude of the jth shape function at time t, an adaptive control strategy has been proposed[4]. Under the assumption that the amplitudes obey uncoupled, linear, ordinary differential equations of known order, but with unknown coefficients, e.g.

$$\frac{d^n}{dt^n} W_j(t) = \sum_{i=1}^{n} [a_{ji} \frac{d^{n-i}}{dt^{n-i}} W_j(t) + b_{ji} \frac{d^{n-i}}{dt^{n-i}} F_j(t)], \tag{2}$$

where $F_j(t)$ are the modal forces, then this strategy[4] combines a truncation of (1) with a modal self-tuning[7] (or simultaneous identification and control) adaptive control algorithm. The modal forces F_j in (2) are given by

$$F_j(t) = \int_{s\varepsilon\Omega} \phi_j(s) f(s, t) ds, \tag{3}$$

where Ω is the set of all system particles and f(s, t) the applied spatially distributed forces. The espoused approach can control the distributed parameter systém of (1) despite a lack of knowledge in the a_{ji} and b_{ji} of (2), which can accomodate inaccurate a priori expansion modeling or variability in the a_{ji} and b_{ji} due to changes in the operating conditions. This approach, to be outlined below, relies on (i) the prespecification of the ϕ_j in (1) yielding uncoupled (2) and (ii) the reasonableness of (1) after truncation. The second assumption will be extensively examined in the latter sections of the paper.

The adaptive modal control strategy[4] can be divided into two stages: one prior to system operation and the other on-line. The recommended steps of pre-activation analysis are:

(i) Determine expansion basis ϕ_j in (1).

(ii) Select finite expansion upper limit to approximate (1).

(iii) Specify sensor locations and relate distributed measurements of d(s, t) to the modal amplitudes $W_j(t)$ in (2) via reversal of (1).

(iv) Determine actuator distribution and formulate effect on modal

forces $F_j(t)$ in (2) via (3).

(v) Establish modal control objectives.

The recommended steps of real-time, adapting, sampled data control formation are:

(vi) Apply previously calculated actuation forces and sense d(s, t) in (1) at sample instant.

(vii) Process sensor data to estimate modal amplitudes $W_j(t)$ in (2).

(viii) Select the modes requiring control.

(ix) Process applied forces f(s, t) via (3) to determine achieved modal forces $F_j(t)$.

(x) Improve the identification of the discretization of (2).

(xi) Design modal controllers on-line with current parameter estimates to meet modal performance objectives.

(xii) Convert desired modal control to actual actuator commands.

(xiii) Repeat (vi)-(xii) at the next sample instant.

The simultaneous identification and control strategy of steps (x) and (xi) could be replaced by a single step improving the controller parameters by direct lumped parameter plant adaptive control[12-14]. The espoused indirect strategy is more intuitive and currently more flexible in terms of control objectives though more restrictive in terms of the general need for plant parameter identifiability[15,16] as manifested in adequate model complexity and sufficiently exciting input requirements.

As outlined the modal controllers rely only on past data to determine the present control action. This allows the considerable (despite the possible parallel execution of steps (x) and (xi))

computation to be done during the full sample period. The steps (vi)-(xii) differ slightly from their earlier description[4] due to a fuller appreciation of the demands of steps (vii) and (ix), especially (vii), which are discussed in later sections. Further, detailed comments on each stage of this step-by-step procedure are available[4]. The next section illustrates this strategy by application to adaptive regulation of the small, linearized, out-of-plane deflection of a large, flexible AMCD.

AMCD Application

Large momentum vectors resulting in the rotation of large space structures can be created smoothly by a solar-powered (and therefore effectively non-depletable), dual momentum vector configuration of two counterrotating AMCDs magnetically attached to the space structure[4,10,11]. The AMCD components of these attitude control devices will probably be as large in diameter as possible for such structures and as small in cross section as possible in order to maximize their momentum/mass ratio. Therefore such AMCDs would behave like lariats with translation, rotation, and deformation modes of disturbance from their nominal planar spinning configuration.

The step-by-step procedure of the preceding section will be followed in designing an adaptive modal controller of such an AMCD.

(i) The boundary conditions of ring closure permit the use of a Fourier series to describe the linearized, out-of-spin-plane deformation of the AMCD

$$d(\theta,\ t) = \sum_{j=0}^{\infty} [W_{cj}(t)\ \cos(j\theta) + W_{sj}(t)\ \sin(j\theta)], \tag{4}$$

which is of the form of (1) with the angle θ, measured around the AMCD ring from a reference ($\theta = 0$) particle to the location in question, as the single spatial variable s. This sinusoidal basis is also the eigenvector basis for this out-of-plane motion of a homogeneous ring[17].

(ii) Assuming that the higher spatial frequency deformations will exhibit lower amplitudes, (4) can be truncated with arbitrary accuracy as

$$d(\theta,\ t) = \sum_{j=0}^{N} [W_{cj}(t)\ \cos(j\theta) + W_{sj}(t)\ \sin(j\theta)]. \tag{5}$$

This limit N may permit accurate approximation of (4) but be an infeasible limit in terms of controller computations. If a further reduction is necessary, (5) and therefore (4) (and in effect (1)) will be roughly approximated by either

$$d_R(\theta,\ t) \triangleq \sum_{j}^{M \text{ of } [0,N]} [W_{cj}(t)\ \cos(j\theta) + W_{sj}(t)\ \sin(j\theta)], \tag{6}$$

where $\sum_{j}^{M \text{ of } [0,N]}$ signifies a summation over the index j where j is any M+1 entries of the set {0, 1, 2, ..., N}, or

$$\hat{d}(\theta,\ t) \triangleq \sum_{j}^{M \text{ of } [0,N]} [\hat{W}_{cj}(t)\ \cos(j\theta) + \hat{W}_{sj}(t)\ \sin(j\theta)], \tag{7}$$

where the $\hat{W}_k$ are not necessarily the corresponding W_k in (6) and (5) due to the selection of $\hat{W}$ to best fit $\hat{d}$ to d given 2M + 1 point measurements of d, e.g. as in the Galerkin approach[1].

(iii) Assume S sensor measurements of ring particle deflections $d(\theta_i, t)$ at $i = 1, 2, \ldots, S$ can be processed simultaneously. These measurements can be decomposed into modal amplitudes W_j by multiple concatenation of (6) (or (7)) with $\sum_j^{M \text{ of } [0,N]}$ replaced by $\sum_{j=0}^{M}$, i.e. for the lower modes, as

$$\begin{bmatrix} d_R(\theta_1,t) \\ d_R(\theta_2,t) \\ \vdots \\ d_R(\theta_S,t) \end{bmatrix} = \begin{bmatrix} 1 & \cos(\theta_1) & \ldots & \cos(M\theta_1) & \sin(\theta_1) & \ldots & \sin(M\theta_1) \\ 1 & \cos(\theta_2) & \ldots & \cos(M\theta_2) & \sin(\theta_2) & \ldots & \sin(M\theta_2) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 1 & \cos(\theta_S) & \ldots & \cos(M\theta_S) & \sin(\theta_S) & \ldots & \sin(M\theta_S) \end{bmatrix} \begin{bmatrix} W_0(t) \\ W_{c1}(t) \\ \vdots \\ W_{cM}(t) \\ W_{s1}(t) \\ \vdots \\ W_{sM}(t) \end{bmatrix}. \quad (8)$$

With appropriate reindexing of the right side of (8) any M modal amplitudes composing d_R in (6) could be written in this matrix form. The θ_i may vary from sample to sample, especially if the sensor(s) is not spinning with the AMCD. Note that (8) requires measurement of d_R not d. Obtaining d_R from d requires observation spillover removal, as will be addressed later.

(iv) Assume A actuators are located in a reference frame fixed with respect to AMCD spin, e.g. the suggested[11] magnetic "bearing" actuators attached to the spacecraft. From (3) where $\Omega = \{\theta | \theta \in [0, 2\pi)\}$ an assumption of point actuation located at σ_i converts the integrals to summations over the set of A actuators. Note that, due to an AMCD spin

rate of Γ radians/second relative to the actuator locations, the σ_i must be converted to the ring particle reference frame via

$$\theta_i(t) = \sigma_i(t) - \Gamma t, \tag{9}$$

which assumes that the references $\theta = 0$ and $\sigma = 0$ were aligned at $t = 0$. Similar to (8), for the C modes to be controlled

$$\begin{bmatrix} F_0(t) \\ F_{c1}(t) \\ \vdots \\ F_{cC}(t) \\ F_{s1}(t) \\ \vdots \\ F_{sC}(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \cos(\theta_1) & \cos(\theta_2) & \cdots & \cos(\theta_A) \\ \vdots & \vdots & & \vdots \\ \cos(C\theta_1) & \cos(C\theta_2) & \cdots & \cos(C\theta_A) \\ \sin(\theta_1) & \sin(\theta_2) & \cdots & \sin(\theta_A) \\ \vdots & \vdots & & \vdots \\ \sin(C\theta_1) & \sin(C\theta_2) & \cdots & \sin(C\theta_A) \end{bmatrix} \cdot \begin{bmatrix} f(\theta_1, t) \\ f(\theta_2, t) \\ \vdots \\ f(\theta_A, t) \end{bmatrix} \tag{10}$$

can be formed.

(v) Ring stabilization requires mode damping. Satisfactory modal damping can be provided by modal pole placement.

(vi) If $d(\theta, t)$ is provided by an effectively instantaneously scanning sensor then any S ring particles can be observed. If the sensors are incorporated with the actuators then (9) must be used since different ring particles will be sensed at each sample instant. One other possibility is more frequent sensor interrogation than actuator reactivation, allowing additional signal processing possibilities before control selection.

(vii) Solution for the $[W_j]$ vector in (8) can be achieved by pseudoinversion[18] or by a DFT[19] if the θ_i are equally spaced. Note that if the θ_i are equally spaced such that $\theta_i = 2\pi i/S$ and $S = 2M$, then $M\theta_i = \pi i$ and the rightmost column of the $S \times (2M + 1)$ matrix in (8) equals zero. Therefore the column of $\sin(M\theta_i)$ entries should be removed to retain invertibility. In such a case the Mth mode sine component is unobservable.

(viii) If $C < M$, the current strategies are to control those C modes either with the lowest spatial frequencies or with the greatest modal amplitudes. The assumption leading to (5) will tend to equate these two classes.

(ix) If $A < 2C + 1$ in (10) then due to the least squares solution implemented in step (xii) of the last sample instant the desired F_j most likely will not have been achieved and (10) must be calculated to determine the forces actually reaching each mode needed in the next step.

(x) For a large, lightweight AMCD (2) will be second order and essentially undamped

$$\frac{d^2}{dt^2}[W_j(t)] + \omega_j^2(\Gamma, t)\, W_j(t) = \left(\frac{1}{M_j}\right) F_j(t), \tag{11}$$

where the M_j denote the effective modal masses[4] and the Γ and t arguments of ω are intended to evoke the slowly time-varying character of the modal amplitude time frequency due to such operating conditions as spin rate and temperature. Note that the magnitude of the oscillatory initial condition response of (11) is inversely proportional to ω_j. The notation

of (11) (and subsequently (12)-(20)) is intended to encompass both W_{cj} and W_{sj}. Note that both W_{cj} and W_{sj} have the same ω_j, i.e. $\omega_{cj} = \omega_{sj}$. Assuming uniform sample intervals of T seconds and constant modal forces over the sample period,

$$W_j(k) = \alpha_{j1} W_j(k-1) + \alpha_{j2} W_j(k-2) + \beta_{j1} F_j(k-1) + \beta_{j2} F_j(k-2) \quad (12)$$

is an exact discretized predictor of the modal amplitude where $\alpha_{j1} = 2 \cos (\omega_j T)$, $\alpha_{j2} = -1$ and $\beta_{j1} = \beta_{j2} = (1 - \cos \omega_j T)/(M_j \omega_j^2)$. The addition of damping in (11) will effect the definitions of α and β but not the form of (12). Note that constant actuator forces f over the sample interval will not generate constant modal forces due to AMCD rotation. This can be combatted by actuator force windowing. If the modal forces vary over the sample period, (12) becomes an approximation only as accurate as the degree of constancy of $F_j(t)$ over $(k-2)T < t < (k-1)T$ and $(k-1)T < t < kT$. If the structure of (12) is used for an adaptive identifier or to structure a direct adaptive controller then uncertainty in ω_j, M_j, and the neglected damping coefficient can be accomodated. The anticipated problem in AMCD control is uncertainty in ω_j. The second-order (12) can be identified by either of two broad classes of recursive parameter estimators termed[20] prediction error and pseudo linear regression and represented by equation error[21] and output error[22] identifiers, respectively. An equation error formulated identifier for (12) of the form

$$\begin{bmatrix} \hat{\alpha}_{j1}(k) \\ \hat{\alpha}_{j2}(k) \\ \hat{\beta}_{j1}(k) \\ \hat{\beta}_{j2}(k) \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_{j1}(k-1) \\ \hat{\alpha}_{j2}(k-1) \\ \hat{\beta}_{j1}(k-1) \\ \hat{\beta}_{j2}(k-1) \end{bmatrix} + \frac{e_j(k-1)}{1+\sum_{i=1}^{2} [\mu_{ji}(k-1)W_j^2(k-i-1)+\rho_{ji}(k-1)F_j^2(k-i-1)]} \cdot$$

$$\begin{bmatrix} \mu_{j1}(k-1)W_j(k-2) \\ \mu_{j2}(k-1)W_j(k-3) \\ \rho_{j1}(k-1)F_j(k-2) \\ \rho_{j2}(k-1)F_j(k-3) \end{bmatrix}, \tag{13}$$

where

$$e_j(k-1) = W_j(k-1) - \sum_{i=1}^{2} [\hat{\alpha}_{ji}(k-1)W_j(k-i-1) + \hat{\beta}_{ji}(k-1)F_j(k-i-1)] \tag{14}$$

and

$$0 < \mu_{ji}(k) \leq \mu_{ji}(k-1) < 2 \quad \text{and} \quad 0 < \rho_{ji}(k) \leq \rho_{ji}(k-1) < 2 \quad \forall\, i,\, k, \tag{15}$$

requires exact measurements of sufficiently rich F_j and W_j for consistent identification.

(xi) Feedback regulation structures of second order dynamic output feedback[4] or equivalently (in the absence of unmeasurable inputs) reconfigured state variable feedback achieved via

$$F_j(k) = \hat{\gamma}_{j1}(k)W_j(k-1) + \hat{\gamma}_{j2}(k)W_j(k-2) + \hat{\delta}_{j1}(k)F_j(k-1) + \hat{\delta}_{j2}(k)F_j(k-2) \tag{16}$$

will cause the modal plant-controller characteristic equation to converge to $z^4 + \lambda_{j1}z^3 + \lambda_{j2}z^2 + \lambda_{j3}z + \lambda_{j4}$ if the controller parameters are chosen

via

$$\hat{\delta}_{j1}(k) = -\lambda_{ji} - \hat{\alpha}_{j1}(k) \tag{17}$$

$$\hat{\gamma}_{j1}(k) = [\hat{\delta}_{j1}(k)\hat{\alpha}_{j2}(k) - \lambda_{j3} + (\hat{\delta}_{j1}(k)\hat{\alpha}_{j1}(k) - \hat{\alpha}_{j2}(k) - \lambda_{j2})$$
$$\cdot(\hat{\alpha}_{j1}(k) - \hat{\alpha}_{j2}(k)\hat{\beta}_{j1}(k)/\hat{\beta}_{j2}(k)) + \lambda_{j4}\hat{\beta}_{j1}(k)/\hat{\beta}_{j2}(k)]/$$
$$[\hat{\beta}_{j1}(k)\hat{\alpha}_{j1}(k) + \hat{\beta}_{j2}(k) - \hat{\alpha}_{j2}(k)\hat{\beta}^2_{j1}(k)/\hat{\beta}_{j2}(k)] \tag{18}$$

$$\hat{\delta}_{j2}(k) = \hat{\alpha}_{j1}(k)\hat{\delta}_{j1}(k) - \lambda_{j2} - \hat{\beta}_{j1}(k)\hat{\gamma}_{j1}(k) - \hat{\alpha}_{j2}(k) \tag{19}$$

$$\hat{\gamma}_{j2}(k) = (\hat{\delta}_{j2}(k)\hat{\alpha}_{j2}(k) - \lambda_{j4})/\hat{\beta}_{j2}(k) \tag{20}$$

and $\hat{\alpha} \to \alpha$ and $\hat{\beta} \to \beta$. Note that the strictly causal formation of $\hat{\alpha}$ and $\hat{\beta}$ in (13)-(14) used in (17)-(20) to parameterize (16) permits the assumed predictive control formation. Furthermore, the form of (16) purposely avoids velocity measurements, which are expected to be difficult to sense for large AMCDs.

(xii) Least-squares solution of (10) for the $f(\cdot,t)$ is required if $A < 2C + 1$.

Simulations

Successful simulations of this AMCD example have been reported elsewhere[4,11,23,24]. Consider here an AMCD described by (5) with $N = 4$ and the ω_j in (11) equal to 0, 2.62, 5.24, 7.85, and 10.47 for $j = 0, 1, 2, 3, 4$ respectively. For an AMCD spin rate of $\Gamma = 60$ degrees/second, a sample period of $T = 0.1$ seconds, and the same damped pole placement objective for each mode of $\lambda_1 = -1.684$,

$\lambda_2 = 1.165$, $\lambda_3 = -0.402$, and $\lambda_4 = 0.0558$, the strategy of the preceding section was applied in the following cases: $N = M = C = 4$, $N = M = 4 > C = 2$, and $N = 4 > M = C = 2$. For each situation the AMCD was given an initial deformation composed of unity modal amplitudes. The adaptive controller was applied, with initial modal frequency estimates $\{\hat{\omega}_j\}_{j=1,\ldots,4} = \{4, 8, 12, 16\}$ converted to initial parameter estimates in (12) via the discretization formulas, in an attempt to stabilize the ring deflections to zero.

The AMCD simulation consisted of (10) with C replaced by N, which converted the applied forces f to modal forces F which were assumed constant over the sample interval, the use of the appropriate F in (12) to update the N modal amplitudes W, and the formation of the deflection d via (5) for all θ. The controller began with measurements of d for use in (8), which were supplied by solution of (5) at the appropriate sensor locations θ. The sensor locations were assumed to be rotating with the AMCD. A special reflective mark on the AMCD and a centrally located, scanning laser detection system can be hypothesized as providing such ring particle deflections. The measured d were used for the d_R in (8) with $M = 4$ or 2 as required. Solution of (8) provided the W used in (13) and (21). The improved parameter estimates provided by (13) with all $\mu = \rho = 1$ were used in (22)-(25) to parameterize (21). When $M > C$, only the F for the modes of lower spatial frequency were calculated. The past F in (21) were provided by the previous results of (21). Step (ix) could be bypassed since A was always chosen equal to or greater than $2C + 1$. Inversion solution

of (10) provided the applied forces f to the AMCD simulation.

Figure 1 with N = M = C = 4 illustrates the anticipated effectiveness of the adaptive strategy in recovering from inaccurate estimates of the ω_j and successfully stabilizing the initial AMCD deflection, eventually to zero displacement. The asterisks on the displacement curves mark the measurement points which are fixed in the ring particle reference frame. The arrowheads along the spatial coordinate θ axis show inertially fixed actuator locations with the length of the arrowshaft proportional to the applied force according to the right-hand scale. The plots are drawn in the ring particle reference frame so the actuator locations appear to regress for the progressing ring. The success in figure 1 is not universal for N = M = C. With only the desired pole locations changed to values nearer the unit circle by $\lambda_1 = -2.314$, $\lambda_2 = 2.429$, $\lambda_3 = -1.176$, $\lambda_4 = 0.2$, the proposed adaptive regulator fails via an often neglected stall mechanism. For example, if the parameters in (13) when used in (17)-(20) lead to an unstable modal controller, W_j^2 will become so large in the denominator in (13) that the parameter estimates are only insignificantly corrected. If this condition persists long enough, all claim of linearity can be abandoned in application, thereby negating claims of eventual identifier convergence. This stall mechanism, a characteristic of simultaneous identification and control not peculiar to just the proposed adaptive DPS control strategy, occurs in the cited example with the W_{c4} (5 seconds) = 4.8×10^4 and growing. This failure can be attributed to the smaller stability margin of higher modal frequency estimates for

low frequency, light damping objectives. Different initial parameter estimates nearer the actual modal frequencies can avoid the stall of this particular example. Also the sufficient excitation requirements of simultaneous identification and control[7] are almost surely not met in this regulation example, thereby encouraging an expectation of failure. However, this pathological case does emphasize a need for an understanding (currently nonexistent even in lumped parameter system adaptive control) of closed-loop singularity migration during simultaneous identification and control.

Figure 2 illustrates the boundedness of the AMCD deflection achieved in adapting from inaccurate ω_j prespecification with $N = M = 4 \geq C = 2$. The visible higher spatial frequency remnants at $t = 15$ seconds are due to the choice to control only the lower spatial frequency components of d, which do decay to zero. Asymptotic regulation of the full deflection is not achieved despite convergent parameter identification in the satisfactorily complex model due to control spillover arising from use of a restricted complexity controller[9]. For lumped-parameter systems this limited success of adaptive control when $N = M > C$ has also been documented elsewhere[25]. For $A > 2C + 1$, i.e. an oversufficient number of actuators for solution of (10), pseudoinverse solution of (10) provides a minimum control f energy solution, which can be expected to reduce the deleterious control spillover. Figure 3 with $A = 9$ for $C = 2$ versus figure 2 where $A = 5$ for $C = 2$ clearly demonstrates this effect.

The possible failure of the proposed adaptive control strategy when $N > M$ is documented in figure 4 where $N = 4 > M = C = 2$. The identifier of (13) is unsuccessful in converging on the actual α and β values due to the use of d and not d_R in (8). Since d is used in (8) and a least squares solution is used to determine the W, then actually (7) and not (6) is applicable. Figure 4, therefore, shows the failure of attempting to use a modal control strategy when approximately matching the full AMCD behavior with a reduced number of modes. For this example, even if the α and β were successfully identified, using the $\hat{W}_j$ and not the W_j in (16) leads to instability. Clearly a mixture of full behavior estimation and reduced-order control strategies, though commonly pursued in practice, is only valid if the modes omitted from the model contribute negligibly to the total behavior. Due to the control spillover reaching the unmodelled modes, even in the event of no initial energy in these mdoes this negligibility can not be assumed. The difficulty adaptive control experiences due to $N > M$ has also been noted in the lumped parameter system case[26]. The next section considers a signal processing strategy to combat this possibly fatal problem by filtering the d of (5) to provide the d_R of (6) to (8).

Observation Spillover Reduction

Inexact sampled W_j are provided to the modal identifiers due to two sources of error: (a) aliasing, both in time and spatial frequency domains, due to discrete measurments of d(s, t) and

(b) reduced-order modeling inaccuracy due to $N > M$. However, based on the characteristic of flexible spacecraft that higher modal frequencies, e.g. the higher frequency ϕ_j for the AMCD in (4) as j becomes large, have correspondingly higher modal amplitude time frequencies, i.e. $\omega_i > \omega_j$ for $i > j$, a strategy for extracting d_R in (6) from measurements of d in (5) has been postulated[24]. This approach assumes (a) satisfactory time-based sampling to avoid aliasing of modal frequencies up to ω_N, i.e. $T < \frac{\pi}{\omega_N}$, (b) a sensor system capable of interrogating any ring particle at any sample instant, (c) band-limiting knowledge of the M sought ω_i, e.g. for the lowest M consecutive ω_i specification of a frequency comfortably between ω_M and ω_{M+1}, and (d) frequency-limited spectra for the F_j leading to separable W_j spectra at desired cutoff points.

For a lowest M frequency approximation, implicit in (8), the following strategy appears reasonable. Assuming equally-spaced sensor measurements, in both space and time, and $S = 2M$, as in point (vii) of the preceding section, a DFT could be used to solve (8); however d_R and not d must be available. If the same ring particle can be measured for deflection at successive sample instants despite AMCD rotation then the sequence $\{d(\theta_i, kT)\}$ over k for a particular i, which is proportional from (5) to a fixed weight sum of the $W_j(kT)$, can be low-pass filtered (LPF) between ω_M and ω_{M+1} removing high time, and therefore also spatial, frequency components leaving $\{d_R(\theta_i, kT)\}$. For each time t composition of (8) is now possible.

The necessary assumptions preceding this particular strategy

description are quite restrictive. Due to the true form of (1) and the possibly tremendous magnitude of N and therefore ω_N (a) is mathematically impossible and only marginally practical. The limitation of (b) is that the sensors can not be colocated with the non-rotating actuators. Even if AMCD spin could be accomodated for DFT purposes the same ring particle could not be sensed at sequential sample instants disallowing the benefit of LPF. Conversely, always measuring the same ring particles could, in certain cases, lengthen the lag time before reaction of the control system to slowly propagating localized disturbances. Assumption (c) may be reasonable for discrete sinusoidal spectra but since (11) is forced by a nonlinearly generated, nonstationary (during adaption) signal in violation of (d), the region of non-overlap between ω_M and ω_{M+1} becomes so small (if not nonexistent) as to require highly refined a priori knowledge. LPF phase distortion must also be assumed negligible. Clearly some higher spatial frequencies will be rejected by the LPF, but due to dissatisfaction of the assumptions pointing to unsatisfactory spectrum separation, d_R will be inexactly obtained. These reservations are even more severe for more complex filtering schemes, such as the effective comb filtering suggested earlier[6] to be achieved via phase locked loops.

One alternative[6] satisfying the (c) and (d) requirements for filtering d_R from d is identification of the free AMCD response. However, this approach does not meet the stated simultaneous identification and control objective. Using a modified gain scheduling

concept[27] to provide a fixed robust control during identification phases is closer to the simultaneous identification and control objective (and may avoid the stall mechanism noted in the previous section) but does spread the W_j spectra again severely limiting the benefits of the LPF. Another seemingly applicable concept is that of adaptive orthogonal filtering[28]. This idea is incorporated by appending to (8) and (13) additional uncontrolled "modes" intended to absorb the spillover effects. These modes would require time-varying dynamic descriptions meaning that the μ and ρ used in estimating their difference equation parameters would need to be significantly larger than those used in estimating the time-invariant modal difference equation parameters. The compensatory ability of these additional model modes seems limited due to the assumptions necessary in adaptive orthogonal filter development. Therefore none of these suggestions appears wholly satisfactory. The conclusion is that currently the observation spillover problem remains unsolved yet requires resolution for broad applicability of the proposed adaptive modal control scheme for DPS.

Conclusion

This paper begins with revision of a previously originated strategy[4] for adaptive modal control of DPS and concludes with the confrontation of the spillover problem, which is extremely severe. In support of the simulation evidence provided it has been proven elsewhere[5] that in the absence of observation spillover and with the use of the eigenvector expansion, adaptive modal control of DPS

is as viable as lumped parameter system indirect adaptive control. It can also be shown[6] why a stable simultaneous identification and control scheme similar to that imbedded in the AMCD example fails in the presence of observation spillover or nonorthogonal expansion. With the necessity of a reduced order model (N > M) the goal of globally stable adaptive DPS controller convergence appears too stringent. Work is in progress to relate observation spillover bounds to parameter identification bounds. Such efforts are directed toward delineating the detrimental influence of observation spillover and the possibility of allowable behavior despite its presence rather than toward its removal.

Suggestions have also been forwarded[29] to remedy the difficulty of ϕ_j selection. For general spacecraft, the fundamental assumption of eigenvector availability for (1) is overly optimistic. Slight inhomogeneities in the AMCD can lead to significant coupling of the Fourier expansion "modes"[30]. Such coupling, if incorporated in the system model, disallows the parallel computation objective for the identifier and "modal" controller. The specter of the necessity to recursively estimate both the basis functions and their associated dynamics raises questions far beyond the current scope of these efforts. However, these are issues that ultimately must be addressed.

The development of an adaptive controller applicable to DPS requires examination of both indirect and direct adaptive control concepts in a necessarily reduced-order model format. Both approaches

are susceptible to spillover degradation. A judicious mixture of robust control, gain-scheduling, on-line versus off-line identification, specific "optimal" objectives versus simpler damping requirements, and local versus global convergence behavior will be required in subsequent efforts. Further research, as in any emerging field, will better identify the weaknesses and strengths of proposed approaches to adaptive control of DPS and uncover additional concerns requiring further original developments.

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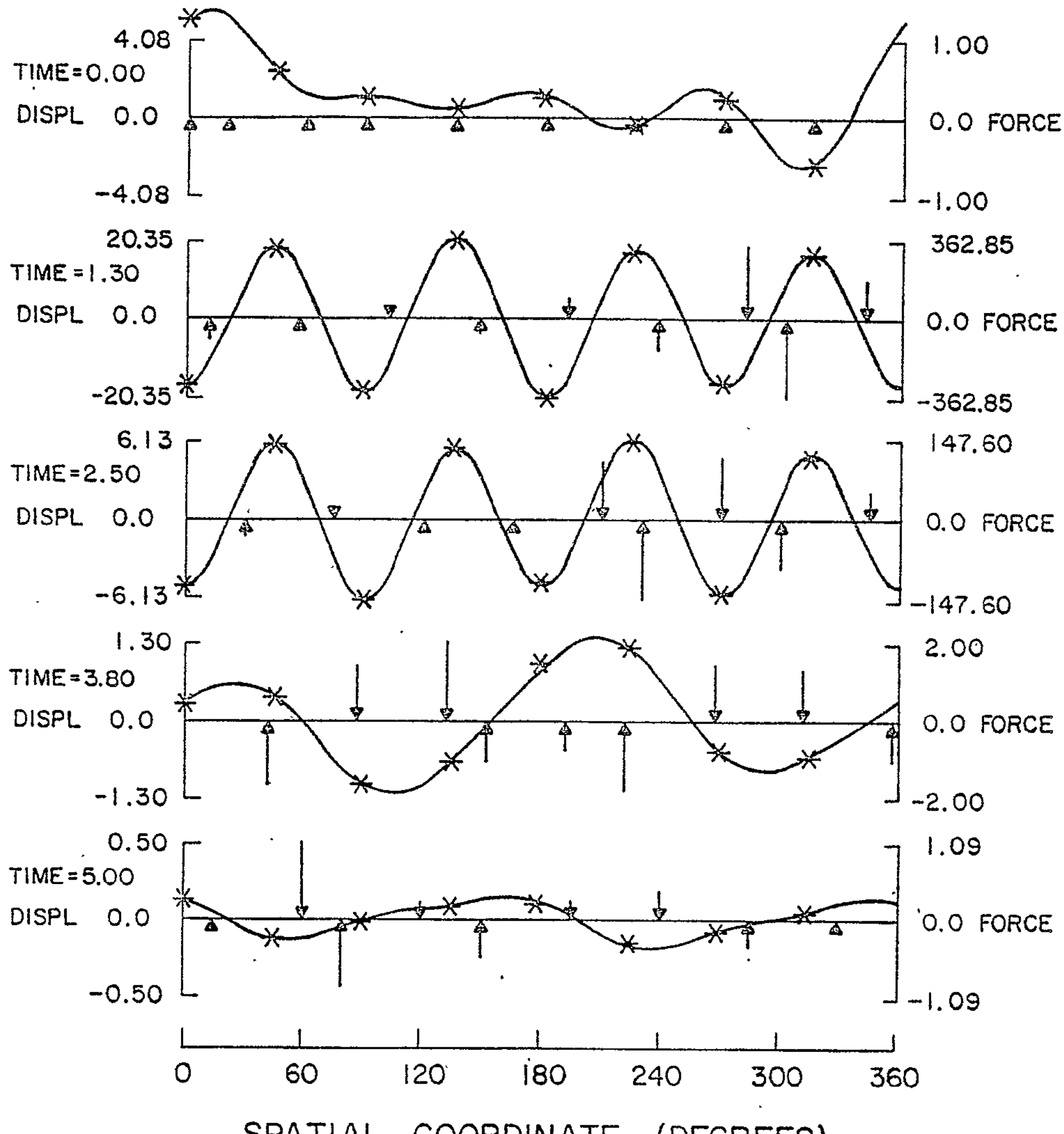
Figure Caption List for C. R. Johnson, Jr. "On Adaptive Modal Control of Large Flexible Spacecraft"

Figure 1: Adaptive Regulation Without Spillover

Figure 2: Adaptive Regulation With Control Spillover

Figure 3: Adaptive Regulation With Control Spillover

Figure 4: Attempted Adaptive Regulation With Control and Observation Spillover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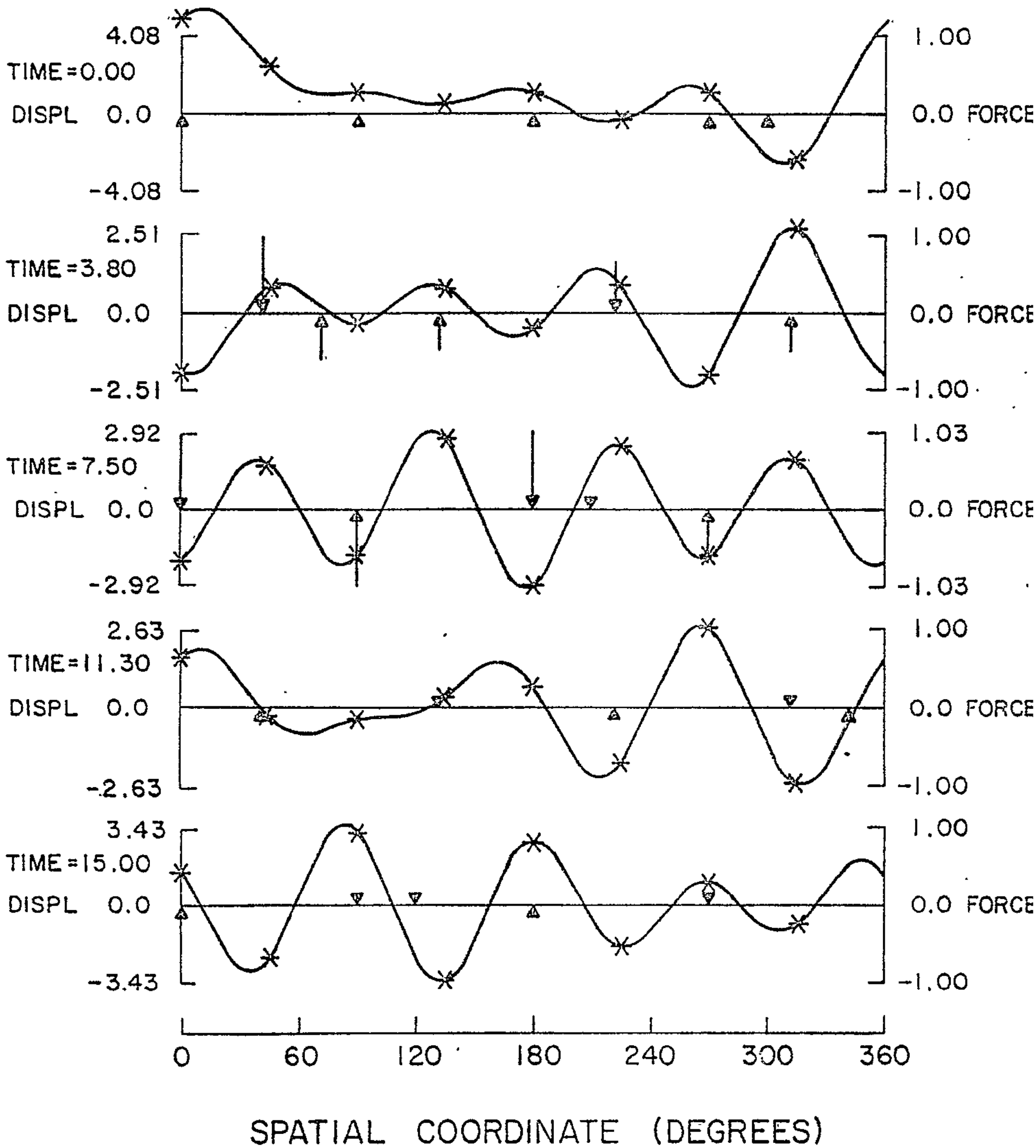

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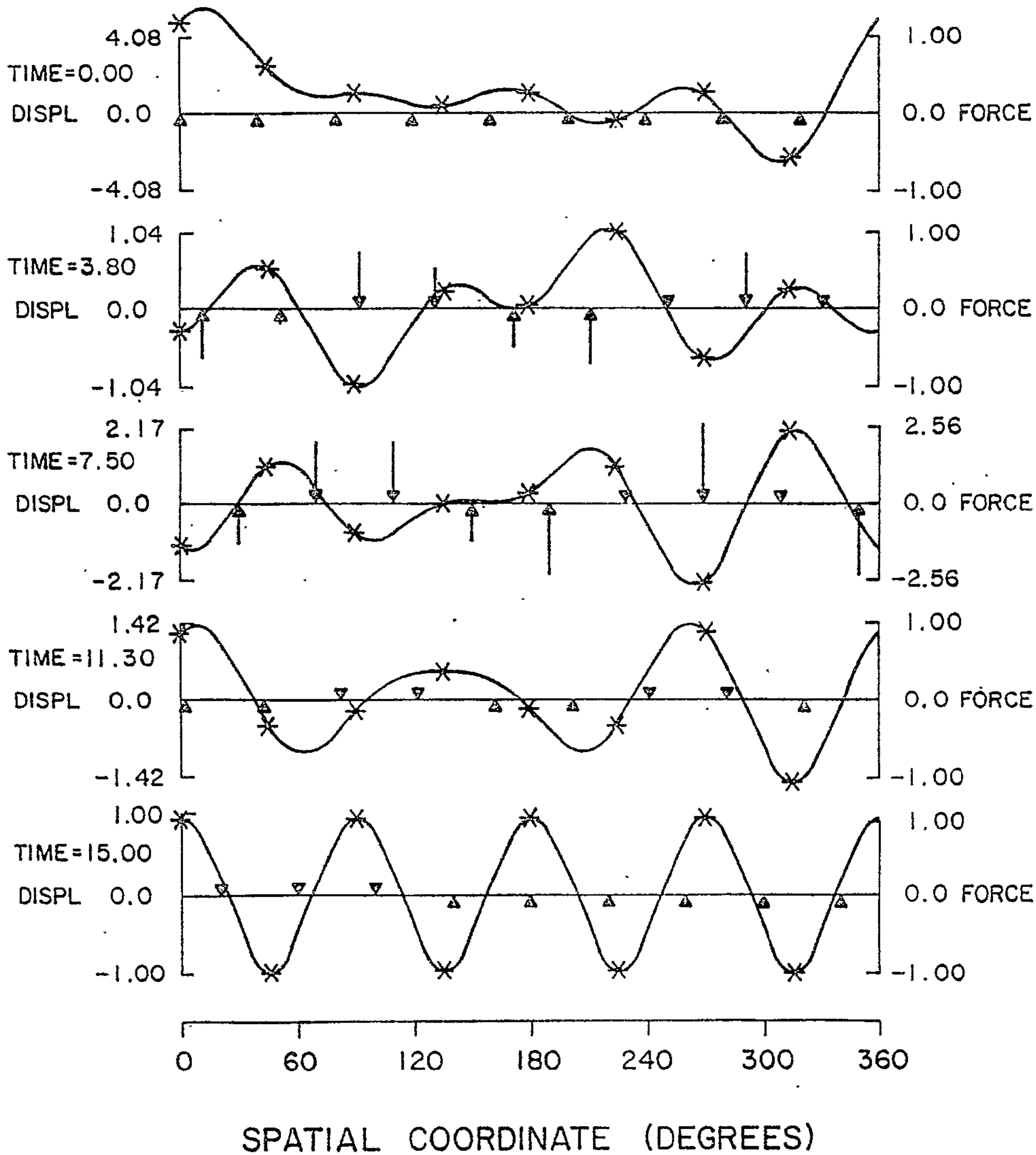

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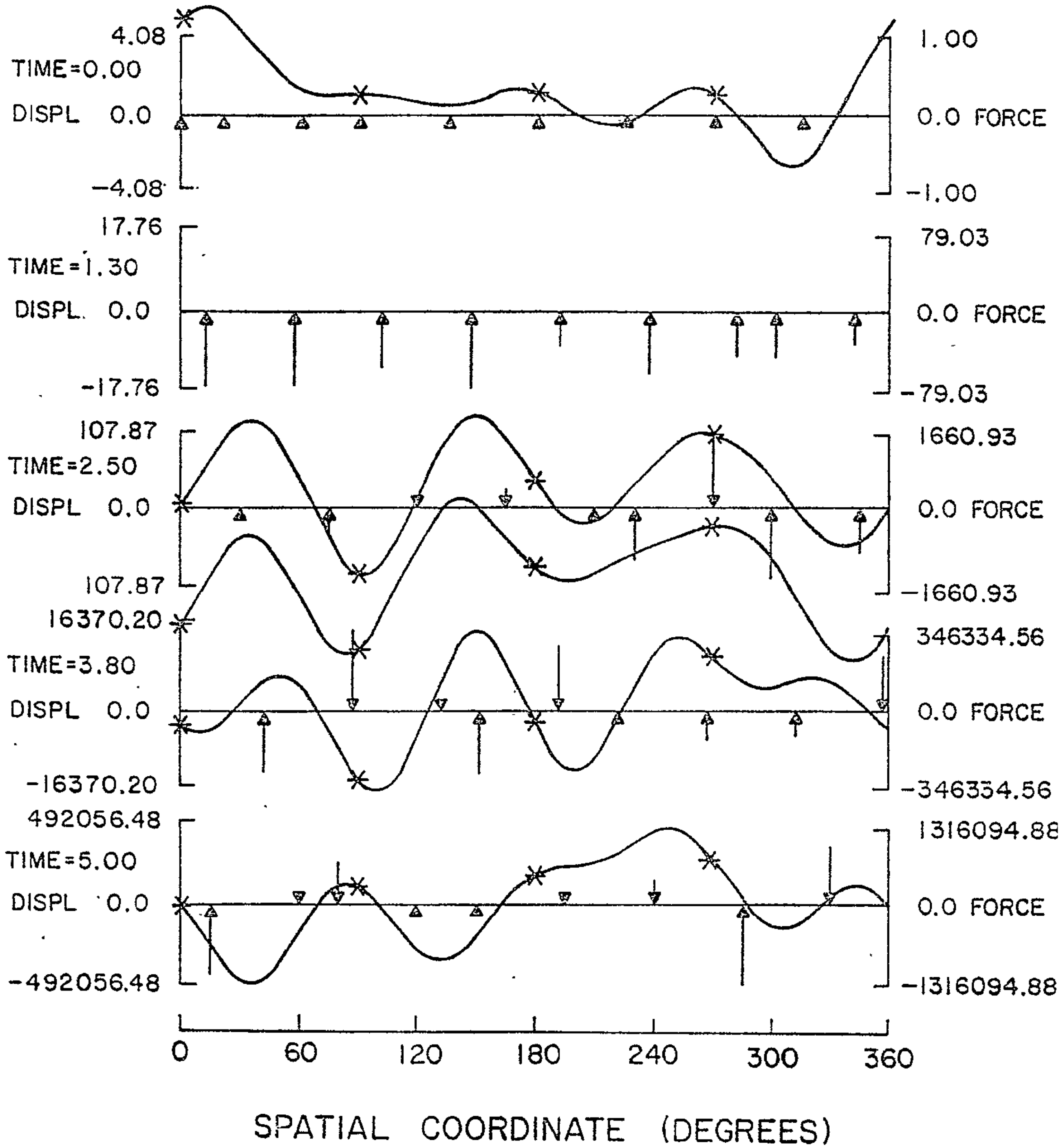
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SPATIAL COORDINATE (DEGREES)

4

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TOWARD ADAPTIVE CONTROL OF LARGE STRUCTURES IN SPACE

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On-line adaptive control is essential for Large Space Structures (LSS) where the modal parameters are poorly known, due to modeling error, or changing, due to variable configurations. It is especially important that such adaptive controllers produce stabilizing controls during adaptation due to the small damping present in LSS. However, any such controller must be based on a reduced-order model of the LSS. The spillover from the unmodelled residuals, as well as the modeling error, can deteriorate the performance of the adaptive controller and, if uncompensated, this spillover can defeat the whole purpose of the adaptive control.

This paper investigates adaptive control for LSS using direct and indirect schemes and points out the mechanisms whereby observation spillover can upset the stability of the controller. The framework for nonadaptive control of LSS is reviewed and many of the generic problems of adaptive LSS control are pointed out within this framework. These generic problems must be overcome for successful operation of adaptive LSS control.

1.0 INTRODUCTION

This paper deals with the basic problems inherent in adaptive control of large space structures (LSS), such as satellites and spacecraft, where the structural parameters are poorly known or slowly time-varying.

With the advent of the Space Shuttle Transportation System, it has become possible to conceive very large spacecraft and satellites which

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could be carried into space and deployed, assembled, or manufactured there. Such LSS would serve a variety of civilian and military needs [1], [2], including electrical energy generation from the solar power satellite - a structure nearly the size of Manhattan Island - to be constructed in space and operated in earth geosynchronous orbit [3]. The control technology needs for such LSS have been discussed in a variety of articles, e.g., [4], [5], and the developing LSS control theory and technology has been surveyed, for example, in [6]-[10].

The size of these structures, their low rigidity, and the small damping available in lightweight construction materials combine to make LSS extremely mechanically flexible. In theory, LSS are distributed parameter systems whose dimension is infinite; however, in practice, their dynamics are usually modeled by large scale systems based on approximate elastic mode data. Active control schemes for LSS are often required to meet stringent requirements for their shape, orientation, alignment, and pointing accuracy. Such active control is limited by the capacity of the on-board control computer, the modeling inaccuracy in current finite element computer codes for analyzing structural dynamics, and available control devices (actuators and sensors); therefore, the controller must be based on some reduced-order model (ROM) of the LSS.

Fundamental problems of LSS control include:

(1) selection of appropriate modes to control for desired system performance;
(2) development of ROM for analysis and controller design;
(3) computation of system model and control parameters;
(4) controller design with multiple distributed actuators and sensors;
(5) the number and location of sensors and actuators for efficient control;
(6) the effect of, and compensation for, residual (unmodelled) modes and modeling error on the closed-loop system performance;
(7) adaptive and self-tuning controllers for LSS with poorly known or changing parameters and configurations.

Item (7) is the basic topic of this paper but it must be considered in the context of the other items with which it is completely intertwined.

The need for adaptive control in LSS arises because of ignorance of the system and changing control regimes. The former occurs as

(a) ignorance of the system structure and order, and (b) ignorance of the system parameters; the latter occurs because of changing configuration of the LSS. Changes in configuration may be due to construction in-space, thermal distortion, or reorientation of subsystems, e.g., rotating solar panels or sunshields; these changes usually produce slowly time-varying parameters. Ignorance of the LSS system structure and order is due to the fundamental problem of modeling a distributed parameter system, e.g., faulty physics, reduced-order models, and ignored nonlinearities; this means that the order of the ROM is lower than that of the actual LSS. Ignorance of the system parameters, while directly related to the system structure, is due to the inherent modeling error present in even the best structural analysis computer codes and to the limitation of testing huge, lightweight LSS on earth; this produces constant but poorly known system parameters. There is a very clear need for an adaptive LSS control methodology that can begin with the best available computed parameters and self-tune its way toward the correct parameters while stablely controlling the LSS and, possibly, adapting to variable configurations.

A great variety of adaptive control schemes exists for lumped parameter, small scale systems [11]; in particular, model reference adaptive methods have achieved a great amount of success in producing stable, convergent adaptive controllers, and adaptive observers for systems whose structure is known and whose parameters are constant but poorly known or slowly time-varying, e.g., [12]-[25]. Adaptive schemes may be direct, i.e., the available control parameters are directly adjusted (adapted) to improve the overall system performance, e.g., [25]-[26], or indirect, i.e., the system parameters are identified (based on the assumed system structure) and the control commands are generated from these parameter estimates as though they were the actual values, e.g., [20], [24], [27].

The abundance of adaptive control methods is overwhelming and an understanding of the interrelationships and structural commonality of these methods is desperately needed; see, e.g., [28], [29], for some beginnings in this direction. Furthermore, the use of such methods on distributed parameter or large scale systems, like LSS, is greatly limited by the ROM problem - the adaptive scheme must be based on a ROM of the actual system

and, hence, the order of the model is, and must remain, substantially lower than the controlled system. In addition, it ~~is~~ seems essential that the LSS adaptive controller provide a stabilizing control in such highly oscillatory systems as LSS.

This paper develops a framework for LSS adaptive control problems and points out generic problems in the use of the most natural direct and indirect adaptive approaches. In other forms, these problems will haunt every use of adaptive control on LSS and must be solved before the valuable benefits of adaptive control can meet the needs of this new application area. A few preliminary attempts at adaptive control for specific distributed parameter systems or LSS have been made in [30]-[36]; also, for the corresponding parameter identification problem for distributed parameter systems, see [37].

2.0 NONADAPTIVE LSS CONTROL

Following [6], the LSS may be described by the partial differential equation:

$$m(x)\, u_{tt}(x,t) + D_0\, u_t(x,t) + A_0\, u(x,t) = F(x,t) \tag{2.1}$$

where $u(x,t)$ represents (possibly, a vector of generalized) displacements of the structure Ω off its equilibrium position due to transient disturbances and the applied force distribution $F(x,t)$. The mass distribution $m(x)$ is positive and bounded on Ω. The internal restoring forces of the structure are represented by $A_0\, u$ where A_0 is an appropriate differential operator with domain $D(A_0)$ defined in a Hilbert space H_0 with inner product $(.,.)_0$. In most ~~product~~ cases, A_0 has discrete spectrum, i.e.,

$$A_0\, \phi_k = \omega_k^2\, \phi_k \tag{2.2}$$

where ω_k are the mode frequencies of vibration and $\phi_k(x)$ are the mode shapes. The damping term $D_0\, u_t$ is generated by an appropriate A_0-bounded differential operator and may represent gyroscopic damping as well as the very small (∿½% critical) natural damping expected in the LSS.

The applied force distribution is given by

$$F(x,t) = F_C(x,t) + F_D(x,t) \tag{2.3}$$

where F_D represents external disturbances and F_C represents the control forces due to M actuators:

$$F_C(x,t) = B_0 f = \sum_{i=1}^{M} b_i(x)\ f_i(t) \tag{2.4}$$

where b_i are the actuator influence functions (usually point devices) and f_i are the control commands. Observations are produced by P sensors:

$$y = C_0 u + C_0' u_t \tag{2.5}$$

where $y_j(t) = (c_j,u)_0 + (c_j',u_t)_0$, for $1 \leq j \leq P$ with c_j being the position sensor influence functions and c_j' the velocity sensor ones (usually point devices).

The state variable form of (2.1) and (2.3)-(2.5) is obtained by taking

$$v(x,t) = [u(x,t),\ u_t(x,t)]^T$$

in $H \equiv D(A_0^{\frac{1}{2}}) \times H_0$ with energy norm:

$$\| v \|^2 = (mu_t,\ u_t) + (A_0^{\frac{1}{2}} u,\ A_0^{\frac{1}{2}} u) \tag{2.6}$$

This produces

$$\begin{cases} v_t = Av + Bf;\ v(0) = v_0 \\ y = Cv \end{cases} \tag{2.7}$$

where we consider the disturbance-free case ($F_D \equiv 0$) and define $B \equiv [0\ B_0]^T$, $C \equiv [C_0\ C_0']$ and $A \equiv \begin{bmatrix} 0 & I \\ -A_0 & -D_0 \end{bmatrix}$. This distributed parameter system is very oscillatory in the sense that the semigroup U(t) generated by A has very little damping:

$$\| U(t) \| \leq M_0\ e^{-\varepsilon t} \text{ for } t \geq 0 \tag{2.8}$$

where $\varepsilon \geq 0$ and small and $M_0 \geq 1$.

The desired performance of the actively controlled LSS greatly effects the design of the controller. Many desirable properties of the active structure can be obtained with constant feedback gains applied to the system state v(x,t); such solutions arise for regulator problems and stabilization (pole placement) problems for LSS. However, the full (infinite

dimensional) state v is never available from a distributed parameter system; only the P sensor outputs y are available.

Implementable controllers for LSS (and most distributed parameter systems) must be based on finite dimensional on-board control computers which process the sensor outputs y and produce control commands f; thus, a reduced-order model (ROM) of the system (2.7) must be used for the controller design. A ROM can be obtained by projecting the system (2.7) in H onto an appropriate finite dimensional subspace H_N; the projections P (onto H_N) and Q (onto the residual subspace) are usually, but not always, orthogonal. Let $v_N = Pv$ and $v_R = Qv$ and, from (2.7), we obtain:

$$\dot{v}_N = A_N v_N + A_{NR} v_R + B_N f \tag{2.9}$$

$$\dot{v}_R = A_{RN} v_N + A_R v_R + B_R f \tag{2.10}$$

$$y = C_N v_N + C_R v_R \tag{2.11}$$

where $A_N = PAP$, $A_{NR} = PAQ$, $B_N = PB$, etc. The terms $B_R f$ and $C_R v_R$ are called control and observation spillover; the terms $A_{NR} v_R$ are called model error. The ROM for this system is given by (2.9) and (2.11) with $A_{NR} = 0$ and $C_R = 0$:

$$\begin{cases} \dot{v}_N = A_N v_N + B_N f \\ y = C_N v_N; \; v_N(0) = Pv_0 \end{cases} \tag{2.12}$$

The ROM state v_N and the residual state v_R form the true system state v with total energy $\|v\|^2$ given by:

$$\begin{cases} v = v_N + v_R \\ \|v\|^2 = \|v_N\|^2 + \|v_R\|^2 \text{ (if projection is orthogonal)} \end{cases}$$

All implementable controller designs based on any ROM must be evaluated in closed-loop with the actual LSS (2.7), and it is in this evaluation that the effects of model error and spillover due to the residuals become apparent.

If the actual mode shapes ϕ_k are known, the modal ROM is a sensible choice:

$$H_N = \{\text{sp } \phi_1 \ldots \phi_N\} \; .$$

and the model error terms A_{NR} and A_{RN} become zero. Of course, any collection of modes could be used; usually, the most easily excited or critical

ones will be chosen. However, in many cases, the partial differential operator A is too complex to provide closed-form mode shapes. Instead finite element approximations of the mode shapes are computed (e.g., via NASTRAN), and these approximate mode shapes can be used to form the ROM for controller design; note that some model error is present when these approximations are used. Henceforth, we will assume the actual mode shapes are available to simplify the discussion but much of our analysis remains valid for approximate mode shapes and other types of ROM.

Modern modal control (MMC) for LSS, as developed in [38], uses the modal (or approximate modal) ROM and develops a controller consisting of a state estimator based on the ROM and a constant gain control law:

$$\begin{cases} \dot{\hat{v}}_N = A_N \hat{v}_N + B_N f + K_N(y-\hat{y}) \\ \hat{y} = C_N \hat{v}_N, \ \hat{v}_N(0) = 0 \end{cases} \tag{2.13}$$

and

$$f = G_N \hat{v}_N \tag{2.14}$$

This controller design requires the ROM (A_N, B_N, C_N) to be controllable and observable for the calculation of control and estimator gains G_N, K_N. These conditions, in modal terms, provide insight into the number and location of actuators and sensors. From [38], (A_N, B_N, C_N) is controllable and observable, when position sensors are used ($C_0 = 0$), if and only if

(1) $\min(P,M) \geq$ max mode frequency multiplicity in the ROM

(2) each sub-block of B_N and C_N associated with a mode frequency ω_N of multiplicity α_N must have rank at least equal to α_N.

Similar results hold for other types of sensors, e.g., velocity, acceleration, or mixtures of types [38], [39]. These results are easy to interpret in terms of the mode shapes, e.g., if no repeated frequencies exist, then the above result says that a single actuator and sensor, not necessarily collocated, will do the job as long as neither is located at any of the ROM mode shape zeros. Since LSS have many symmetries and rigid body modes, it is not often that a LSS control problem will have a

controllable observable ROM with only one pair of devices; this has consequences for the adaptive control problem to be discussed later.

Let the estimator error $e_N \equiv \hat{v}_N - v_N$ be defined and, from eqs. (2.9)-(2.11) and (2.13)-(2.14), obtain

$$\dot{v}_N = (A_N + B_N G_N) v_N + B_N G_N e_N \tag{2.15}$$

$$\dot{e}_N = (A_N - K_N C_N) e_N + K_N C_R v_R \tag{2.16}$$

$$\dot{v}_R = B_R G_N v_N + B_R G_N e_N + A_R v_R \tag{2.17}$$

This shows the effect of spillover on the closed-loop system: even though $A_N + B_N G_N$, $A_N - K_N C_N$, and A_R are stable, the closed-loop system need not be stable. When either control or observation spillover is absent ($B_R = 0$ or $C_R = 0$), then stability is assured; otherwise, spillover causes pole-shifting and can induce instabilities [38], [40]. Bounds on the destabilizing effect of observation and control spillover were produced in [38] and can be extended to the case where some model error and small nonlinearities are present [41]. Such bounds give an indication of how much spillover the closed-loop system can tolerate.

A variety of methods have been suggested to reduce spillover [6]. One obvious way would be to prefilter the sensor outputs, with a bandpass filter, to substantially reduce observation spillover. This alleviates the worst pole-shifting problem; bounds on system performance with control spillover alone can be found in [42]. Note that the post-filter of the controller outputs could do the same job by reducing control spillover; this interchange of filter and controller is possible due to linearity and time-invariance. The trade-off with this means of reducing spillover is that the prefilter introduces phase distortion which can have a destabilizing effect of its own. Therefore, a very high order filter may be required to keep the phase distortion acceptable; phase-locked-loop quadrature filters may be another solution [43]. Even in nonadaptive LSS control, the spillover and model error problem is a fundamental one.

Finally, we should note in this section that digitally implemented controls would be based on discrete-time versions of the distributed parameter system (2.7). One such version is obtained by using a uniform time step Δt:

$$\begin{cases} v(k+1) = \Phi v(k) + E\, f_k \\ y(k) = Cv(k) \end{cases} \tag{2.18}$$

where $\Phi \equiv U(\Delta t)$ and $E \equiv E_0\, B = \int_0^{\Delta t} U(\tau)\, d\tau\, B$ and the control command is a constant f_k over the interval $(k - 1)\,\Delta t \leq t < k\,\Delta t$. Other versions of this could be obtained with nonuniform time steps. When the ROM procedure of projecting onto the subspace H_N is used, we obtain the discrete-time versions of (2.9)-(2.11):

$$v_N(k+1) = \Phi_N\, v_N(k) + \Phi_{NR}\, v_R(k) + E_N\, f_k \tag{2.19}$$

$$v_R(k+1) = \Phi_{RN}\, v_N(k) + \Phi_R\, v_R(k) + E_N\, f_k \tag{2.20}$$

$$y(k) = C_N\, v_N(k) + C_R\, v_R(k) \tag{2.21}$$

When H_N is the modal subspace, the above become:

$$v_N(k+1) = \Phi_N\, v_N(k) + E_0\, B_N\, f_k \tag{2.22}$$

$$v_R(k+1) = \Phi_R\, v_R(k) + E_0\, B_R\, f_k \tag{2.23}$$

$$y(k) = C_N\, v_N(k) + C_R\, v_R(k) \tag{2.24}$$

where (2.22) is the same as that obtained by directly discretizing the ROM in (2.12); if the exact mode shapes are not available, these two discretizations may yield different results. In addition, the sampling process can alias residual modes and increase observation spillover and the zero-order hold process can spread-out the control command signal spectrum and, hence, increase control spillover by increasing the energy in the residual mode spectrum; this has been observed and investigated in [44]. Therefore, the time discretization is a very important factor in the design of implementable LSS controllers.

3.0 TOWARD ADAPTIVE CONTROL OF LSS

In order to design MMC, or other controllers, for LSS as proposed in the previous section, we must have knowledge of the ROM parameters (A_N, B_N, C_N). These parameters are obtained from modal data; they are the mode frequencies for A_N and the mode shapes at actuator and sensor locations for B_N and C_N, respectively. This data is required for three reasons:

(1) to determine controllability and observability of the ROM and, hence, to help locate control devices effectively;
(2) to design control and estimator gains;
(3) to use in the state estimator's internal model.

However, we have noted in Sec. 1.0 the sources of error for this data; consequently, a need arises for an adaptive version of the MMC of Sec. 2.0.

The most logical and reasonable procedure to obtain adaptive controllers for a LSS seems to be the following:

<u>Procedure for Adaptive LSS Control</u>

(a) choose a "nice" reduced-order model (ROM); a modal ROM would be the obvious choice;
(b) use your "favorite" lumped parameter adaptive control scheme;
(c) design the adaptive controller as though the ROM were the actual LSS to be controlled, i.e., ignore the unmodeled residual part of the structure;
(d) use this adaptive controller in closed-loop with the actual LSS and hope for the best.

There is nothing wrong with following this procedure as a best first guess - in a way, there is little else that one can do to produce an implementable adaptive LSS controller.

In some cases, spillover is sufficiently small or enough other mathematical structure is present in the system, e.g., a high level of damping in the distributed parameter system [31] or low level of performance required from the controller (increased damping via direct velocity feedback) [32], to allow the adaptive controller to operate. However, these situations are rare with LSS and one would not like to count on the "generosity of nature" or the temporary suspension of Murphy's Law as part of the above design procedure. Consequently, we would add the following items to that procedure:

(e) analyze computer simulations of higher-order models of the LSS in closed-loop with the adaptive controller based on the lower-order ROM (e.g., simulate more modes than you plan to control);

(f) investigate the specific mathematical mechanisms whereby the residual (unmodeled) part of the LSS couples into a given adaptive control scheme (e.g., find out where and how spillover affects the adaptive controller);

(g) obtain mathematical results on the amount of spillover and/or model error that can be tolerated in the closed-loop system and still provide adequate adaptation and control;

(h) develop spillover and model error compensation schemes to augment the adaptive controller when the residuals cannot be tolerated (e.g., when the conditions of (g) are not satisfied);

(i) recheck (g) with this compensation in the closed-loop system.

We believe that, within the basic framework of LSS control as described in Sec. 2.0, this Augmented Procedure (a)-(i) will go a long way toward revealing the problems of adaptive LSS control (and indeed, most other adaptive control situations where control must be based on a ROM) and will help to focus needed attention on these crucial issues. For example, although Step (e) would be done most likely at some point in the system development phase of any project as complex as the construction and operation of a LSS, often it is done much too late and the design is "set in concrete (or in this case, graphite-epoxy)"; the other steps (f)-(i) may not be done at all. Yet, ignoring the effects of the residual unmodeled LSS can produce some disasterous behavior in the adaptive controller; this was pointed out quite clearly in the LSS example in [36].

When an adaptive scheme is applied to such a lightly damped, oscillatory system as a LSS, the stability of the closed-loop system during adaptation is a necessity; therefore, we do not view convergence and (global) stability results as luxuries for adaptive LSS control and shall only consider appropriate those lumped parameter adaptive schemes for which such results are available. However, even a globally stable adaptive scheme may prove to be unstable when it is used in closed-loop with the actual LSS instead of the ROM on which it was based. This is not a failure of the adaptive scheme; it is a failure to satisfy the mathematical hypothesis of the stability result associated with the scheme.

In the rest of this section, we shall study the use of two very general adaptive schemes which seem to illustrate the problems and potential of adaptive LSS control: the indirect schemes of [23]-[24], which use an adaptive observer and operate in continuous time, and the discrete-time, direct or indirect, schemes which are based on an Autoregressive Moving Average (ARMA) model of the controlled plant, e.g., [25]-[27]. These approaches represent a good cross-section of available lumped parameter adaptive control schemes which have been shown to possess the desired stability properties. We emphasize that the point of this section is not to criticize or slander these schemes; rather, we mean to point out where the hypotheses of their stability results are violated, and must be modified, when we attempt to use them on LSS. We feel that consideration of these approaches within the context of the Augmented Procedure for Adaptive LSS Control (a)-(i) will illustrate the generic difficulties in the application of existing, well-behaved, lumped parameter adaptive control schemes to LSS.

3.1 Multivariable Systems Converted to Scalar Systems

The results of many stable adaptive schemes, e.g., [13]-[14], [23]-[24], are limited to a single actuator and/or a single sensor; yet, we have seen in Sec. 2.0 that most LSS control problems will involve multiple actuators and sensors. One way to deal with this (although, admittedly it has its drawbacks) is to convert the controllable observable LSS problem via output feedback into one that is controllable and observable from a single actuator and/or single sensor; this can be done with almost any output feedback gains [45]-[47]. These gains would have to be based on the best available calculated ROM data and the designer must hope that they will continue to do their job during adaptation.

The output feedback modifies the original system (2.9)-(2.11) to become:

$$\dot{v}_N = (A_N + B_N H_N C_N)\, v_N + (A_{NR} + B_N H_N C_R)\, v_R + b_N f \tag{3.1}$$

$$\dot{v}_R = (A_{RN} + B_R H_N C_N)\, v_N + (A_R + B_R H_N C_R)\, v_R + b_R f \tag{3.2}$$

$$y = c_N^T v_N + c_R^T v_R \tag{3.3}$$

where H_N is the output feedback gain matrix, b_N, b_R, c_N, c_R are vectors,

and f, y have been renamed. Let $A_N + B_N H_N C_N$ be $\overline{A}_N$, etc. and we have that the new ROM $(\overline{A}_N, b_N, c_N^T)$ is a controllable, observable single input, single output system and (3.1) and (3.2) become:

$$\dot{v}_N = \overline{A}_N v_N + \overline{A}_{NR} v_R + b_N f \tag{3.4}$$

$$\dot{v}_R = \overline{A}_{RN} v_N + \overline{A}_R v_R + b_R f \tag{3.5}$$

3.2 Indirect Adaptive Controller Design

We apply the design of the adaptive controller in [24] directly to the ROM consisting of (3.4) and (3.3) with the assumption, for now, that

$$\overline{A}_{NR} = 0 \text{ and } c_R^T = 0.$$

The control law is given by

$$f(t) = g_N^T \hat{v}_N(t) + f_c(t) \tag{3.6}$$

where g_N is a <u>constant</u> gain vector, f_c is a "sufficiently rich" external signal (more about this later), and $\hat{v}_N$ is derived from the following adaptive observer (or state estimator):

$$\dot{\hat{v}}_N = F \hat{v}_N + gy + hf; \; \hat{v}_N(0) = \hat{v}_0 \tag{3.7}$$

where F is an arbitrary, stable matrix and g, h are unknown parameter vectors. The appropriate <u>matching conditions</u> are:

$$\begin{cases} F + G^* c_N^T = \overline{A}_N \\ h^* = b_N \\ \hat{v}_0 = \hat{v}_N(0) \end{cases} \tag{3.8}$$

where g*, h* are constant.

Let $p_0^* \equiv [g^{*T} \; h^{*T} \; v_N(0)^T]^T$ and note that

$$\dot{p}^* = \tilde{F} p^*; \; p^*(0) = p_0 \tag{3.9}$$

where $\tilde{F} \equiv \text{diag}\,[0 \; 0 \; F]$ and, when $g = g^*$, $h = h^*$, we have from (3.8) and (3.7):

$$v_N(t) = [M(t) \; I_N] \, p^*(t) \tag{3.10}$$

where $M(t) \equiv \int_0^t e^{F(t-\tau)} [I_N \, y(\tau) \; I_N \, f(\tau)] \, d\tau$ and we have used the fact that h, f are scalars.

Now (3.7) can be rewritten as

$$\hat{v}_N(t) = [M(t)\ I_N]\ p(t) \tag{3.11}$$

$$\dot{M}(t) = F\ M(t) + [I_N\ y(t)\ I_N\ f(t)] \tag{3.12}$$

$$M(0) = 0 \tag{3.13}$$

where M(t) is as defined in (3.10), and we have yet to produce the adaptive law to generate p(t). This adaptive law is given by the following:

$$\dot{p}(t) = \tilde{F}\ p(t) - \alpha(t)\ [R(t)\ p(t) + r(t)] \tag{3.14}$$

where $p(0) = p_0$ is arbitrary and

$$\begin{cases} \dot{R}(t) = -q\ R(t) - \tilde{F}^T\ R(t) - R(t)\ \tilde{F} + [M(t)\ I_N]^T\ c_N\ c_N^T\ [M(t)\ I_N] \\ R(0) = 0 \end{cases} \tag{3.15}$$

and

$$\begin{cases} \dot{r}(t) = -q\ r(t) - \tilde{F}^T\ r(t) - [M(t)\ I_N]^T\ c_N\ y(t) \\ r(0) = 0 \end{cases} \tag{3.16}$$

where $\alpha(t)$ is the adaptive gain and the constant q is chosen to exceed twice the absolute value of the real parts of the eigenvalues of F. The adaptive gain is chosen so that

$$\alpha(t) = \gamma + ||\dot{\mu}(t)|| \tag{3.17}$$

where

$$\dot{\mu}(t) = -\lambda\ \mu(t) + (N/2)^{1/2}\ (|y(t)| + |f(t)|) \tag{3.18}$$

with λ positive and $F + F^T \leq 2\lambda\ I_N$.

3.3 Convergence Results: What Goes Wrong?

All of the above is exactly as stated in [24] where it is also shown in Appendix I and II that

$$\mu(t) \geq ||M(t)|| \tag{3.19}$$

and, with f(t) sufficiently rich in frequencies, there is a t_1 such that

$$R(t) \geq \rho\ I_N > 0 \text{ for all } t \geq t_1 \tag{3.20}$$

In addition, it is shown in Appendix III that

$$R(t)\ p^*(t) + r(t) \equiv 0 \tag{3.21}$$

This is very crucial to the stability results of [24] and it is here that observation spillover (i.e., the fact that we are using a ROM) appears - (3.21) is not valid when $c_R \neq 0$; however,

$$R(t)\ p^*(t) + r(t) = \Delta_R(t) \tag{3.22}$$

where $\Delta_R(t) \equiv \int_0^t e^{-(q + \tilde{F}^T)\ (t - \tau)}\ [M(\tau)\ I_N]^T\ c_N\ c_R^T\ v_R(\tau)\ d\tau$

Let $e_N(t) \equiv \hat{v}_N(t) - v_N(t)$ and $\Delta p(t) \equiv p(t) - p^*(t)$; then

$$e_N(t) = [M(t)\ I_N]\ \Delta p(t) \tag{3.23}$$

and

$$\Delta\dot{p}(t) = [\tilde{F} - \alpha(t)\ R(t)]\ \Delta p(t) - \alpha(t)\ \Delta_R(t) \tag{3.24}$$

Consider $V(t) \equiv \Delta p(t)^T\ \Delta p(t)$ and we obtain:

$$\dot{V}(t) \leq -\gamma\ \rho\ V(t) - 2\gamma\ \Delta_R(t)\ \Delta p(t) \tag{3.25}$$

where $V(0) = \Delta p_0^T\ \Delta p_0$ and $\Delta p_0 = p_0 - p_0^*$.

This follows [24] except for the additional term in (3.25); also note that

$$||e_N(t)||^2 \leq V(t)\ [1 + \mu(t)] \tag{3.26}$$

Let $\Delta v_N(t) \equiv v_N(t) - v_N^*(t)$ and $\Delta v_R(t) \equiv v_R(t) - v_R^*(t)$ where v_N^* and v_R^* represent the ideal states of (3.4)-(3.5) when the parameters are exactly known:

$$\dot{v}_N^* = (\bar{A}_N + b_N\ g_N^T)\ v_N^* + \bar{A}_{NR}\ v_R^* + b_N\ f_c \tag{3.27}$$

$$\dot{v}_R^* = (\bar{A}_{RN} + b_N\ g_N^T)\ v_N^* + \bar{A}_R\ v_R^* + b_R\ f_c \tag{3.28}$$

with $v_N^*(0) = v_N(0)$ and $v_R^*(0) = v_R(0)$.

When the implementable adaptive control law (3.6) is used, we obtain:

$$\Delta\dot{v}_N = (\bar{A}_N + b_N\ g_N^T)\ \Delta v_N + \bar{A}_{NR}\ \Delta v_R + b_N\ g_N^T\ e_N \tag{3.29}$$

$$\Delta\dot{v}_R = (\bar{A}_{RN} + b_N\ g_N^T)\ \Delta v_R + \bar{A}_R\ \Delta v_R + b_R\ g_N^T\ e_N \tag{3.30}$$

and we have the following result:

THEOREM 3.1: Assume

(1) $c_R = 0$
(2) $\bar{A}_N + b_N g_N^T$ stable
(3) $\bar{A}_R$ stable
(4) $\bar{A}_{NR} = 0$
(5) $f_c(t)$ sufficiently rich (i.e., it has at least 3N/2 distinct frequencies)

Then there is a $\delta > 0$ such that for all $||\Delta p_0||^2 \leq \delta$:

(a) $e_N(t)$ is bounded and (eventually) vanishes with an arbitrary exponential rate
(b) $\lim_{t\to\infty} \Delta v_N(t) = 0$
(c) $\lim_{t\to\infty} \Delta v_R(t) = 0$.

Therefore, even though the closed-loop system with the adaptive controller is highly nonlinear, it is stable while the adaptation is taking place. In particular, (1), (3), and (4) are satisfied if there is no observation spillover ($C_R = 0$) and no model error ($A_{NR} = 0$) and some damping in the residuals (i.e., A_R is stable).

Proof: This result follows from the results of [24] because $\Delta_R(t) \equiv 0$ in (3.22) when $c_R^T = 0$. The stability of (3.29)-(3.30) is determined by $\bar{A}_N + b_N g_N^T$ and $\bar{A}_R$ when $\bar{A}_{NR} = 0$. Also, if $C_R = 0$, then $\bar{A}_{NR} = A_{NR}$, $\bar{A}_R = A_R$, and $c_R^T = 0$ since it is a row vector of C_R. #

Note that the stability of the system with adaptive control is determined by that of (3.27)-(3.28) - the ideal case where the parameters are known and the external signal f_c is applied. This is natural, since adaptation cannot take place without f_c present; however, after adaptation, we would most likely want to turn off f_c. In addition, we could choose the f_c signal so as not to excite the residual frequencies whenever sufficient spectral separation is present. Still, we would need to turn on f_c now and then, in order to "tune-up" the controller.

3.4 Spillover Compensation for the Indirect Adaptive Controller

The above result is merely a slight extension of the results of [24] to a special case of the adaptive controller based on an ROM instead of

the full system. However, it does suggest that some form of compensation should be used to eliminate the observation spillover. Such compensation must be essentially independent of the parameters of the ROM; yet, most methods of spillover reduction require knowledge of the ROM (and some residual) parameters.

One approach to spillover compensation already suggested in Sec. 2.0 is prefiltering the sensor outputs to remove or greatly reduce the term $C_R\, v_R(t)$. Such prefiltering can be achieved with low-pass or band-pass filters when the ROM frequencies are known and separated from the residual frequencies. However, the modal frequency data is part of the poorly known parameter data.

In an attempt to resolve this predicament, we could try using phase-locked loop (PLL) based filters with the center frequency of each loop tuned to the best approximation available of the corresponding ROM data. The PLL will adapt itself until it tracks the actual mode frequency and, after "lock-on," it will behave as a narrow band-pass, linear filter which tunes out the observation spillover from the other frequencies [43]. Of course, sufficient spectral separation must be present, the calculated values of the ROM modal frequencies must be sufficiently good, the distortion introduced by the filter must be sufficiently small, and the adaptive controller must not shift the poles around too much. Thus, the PLL prefilter is not a panacea! But, it might work to reduce spillover and, if it does, it has the added advantage that its output could also reveal better estimates of the modal frequencies; this would take some of the load off the adaptive observer. If the adaptation mechanism causes too much pole shifting, the ROM frequencies could be excited via f_c and identified in open-loop by the PLL filters before the adaptive controller is turned on. Note that modal frequency data is usually better known, via computer approximation, than modal shape data; hence, this approach might not be unreasonable. Another possible, but untried, approach to spillover compensation might be an adaptive version of the orthogonal filter in [48].

Note that some prefiltering (and postfiltering) always takes place due to the bandwidth limitations of the sensors (and actuators). Whether

A problem that arises with the use of the results in [24] and their modification to LSS is that the constant feedback gains g_N^T must be calculated, in advance, to stabilize $\overline{A}_N + b_N g_N^T$. It would be better if these gains were adapted along with the parameters in the observer. Of course, after adaptation has taken place, they could be recalculated from the "tuned-up" parameters, but, in some cases, the adaptation phase is never over, e.g., slowly-varying parameters. Other approaches could be used for adaptive pole-placement, e.g., [20], [21], but these also have their limitations.

3.5 ARMA-Gettin'

Many discrete-time adaptive control schemes depend on an Auto-Regressive Moving Average (ARMA) representation of the plant in discrete-time, e.g., [25]-[27]:

$$y(k + N) = \sum_{r=1}^{N} \alpha_r\, y(k + r - 1) + \sum_{r=1}^{N} \beta_r\, f(k + r - 1) \qquad (3.31)$$

for some N and appropriate matrices α_r, β_r. What the ARMA says is that, after N time steps, the present output is only related to the past N outputs and inputs. Existence of an ARMA is directly related to the finite dimensionality of the plant (N is usually that dimension) and is obtained using the Cayley-Hamilton theorem for matrices. For LSS, only a "<u>quasi-ARMA</u>" can exist; these were considered in detail in [49]. From the Appendix of [49], we obtain the quasi-ARMA for the LSS (2.18) or (2.19)-(2.21):

$$y(k + N) = \sum_{r=1}^{N} \alpha_r\, y(k + r - 1) + \sum_{r=1}^{N} \Gamma_r\, E_N\, f(k + r - 1) + R(k) \qquad (3.32)$$

where

$$R(k) = C_R\, v_R(k + N) + \sum_{r=1}^{N} \Delta_r\, v_R(k + r - 1)$$

$$\Delta_r = \Gamma_r\, \Phi_{NR} - \alpha_r\, C_R$$

and Γ_r is defined in the Appendix of [49]. Since $R(k) = 0$ when $C_R = 0$ and $\Delta_r = 0$, we have the following result:

THEOREM 3.2: When the observation spillover (C_R) and the model error term (Φ_{NR}) are both zero, the quasi-ARMA (3.32) is a true ARMA for the LSS (2.18), and any stable adaptive scheme based on this ARMA will be globally stable when used in closed-loop with the actual LSS (2.18).

When the rather stringent hypothesis of Theo. 3.2 is not satisfied (as it may not be in practice), any adaptive LSS control scheme based on the quasi-ARMA (3.31) must ignore R(k) in order to be implementable. However, R(k) is the term where the residual effects - spillover and model error - enter the scheme and can cause instability. Again, as in Sec. 3.4, prefiltering or other compensation might be tried in an attempt to reduce or eliminate this term.

4.0 CONCLUSIONS

In an attempt to point out the crucial issues and generic problems associated with adaptive control of large aerospace structures (LSS), we have reviewed the framework for nonadaptive LSS control (Sec. 2.0) and, within this framework, have proposed a general procedure, based on reduced-order models (ROM) of the LSS, for obtaining and assessing the problems of adaptive LSS controllers (3.0). In addition, we have taken a closer look at the use of certain well-known, lumped parameter, stable adaptive control schemes in this procedure. Taking these schemes as representative of the basic ideas present in all lumped parameter adaptive control approaches, we have obtained corresponding LSS adaptive controllers and found the following generic problems associated with adaptive LSS control:

(1) LSS are distributed parameter or large scale systems; therefore, the plant dimension is always larger than the dimension of the adaptive controller, which must be based on a ROM;

(2) LSS control must often be done with more than one actuator and sensor; conversion of multivariable to scalar systems via output feedback introduces problems of its own (e.g., stability of the residuals);

(3) LSS control problems are often non-minimum phase due to noncollocated actuators and sensors;

(4) Interaction of the residuals with the adaptive controller may negate the stabilizing properties of the controller due to observation spillover; this interaction is much worse due to the nonlinear nature of adaptive control;

(5) Methods of spillover compensation for LSS often require knowledge of the ROM (and some residual) parameters - the very data that are poorly known;

(6) The adaptation mechanism may shift the closed-loop frequencies around; this counteracts the benefits of any prefiltering unless sufficient spectral separation is maintained;

(7) Indirect adaptive controllers need sufficient excitation from an external signal f_c; however, this signal may substantially excite the residuals.

(8) Discrete-time adaptive controllers can only be based on quasi-ARMA rather than strict ARMA representations of the LSS; this may negate the stability properties of such a controller.

Stable adaptation is essential for such highly oscillatory systems as LSS, yet our preliminary stability results, Theos. 3.1 and 3.2, both require that observation spillover be somehow completely eliminated before it reaches the adaptive control logic; certainly, this is not an easy thing to do in general! Perhaps, global stability is too much to ask for LSS adaptive control because observation spillover will always be present to some degree in LSS control. However, it seems reasonable to hope for the development of spillover bounds to give some idea of regions of stability for the successful operation of LSS adaptive

control. Some comparison should be made between stable adaptive controllers based on ROM and stable, robust control schemes, e.g., [50], [51].

In closing, we would like to say that it is not our intent to present a gloomy picture for the application of adaptive control to LSS. In fact, the need for adaptive control in LSS is already becoming quite clear, and recognition of this need comes, for a change, at an appropriate time - before any LSS have been built and put into space. However, the development of adaptive control for LSS will not take place overnight and will not be done by one or two people. Consequently, what we have tried to stress here for the interested researcher are some of the fundamental problems that arise and the basic steps which need to be taken toward the goal of successful adaptive control of LSS. In the long run, we have high hopes for the success of this endeavor and we expect that adaptive control theory will profit by its association with large aerospace structures, as well.

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